Further development of various equations concerning the Ramanujan Manuscript Book 1. New possible mathematical connections with various topics regarding the Dark Matter, the String Theory, in particular the Supersymmetry Breaking. XI

Michele Nardelli[1], **Antonio Nardelli**[2]

Abstract

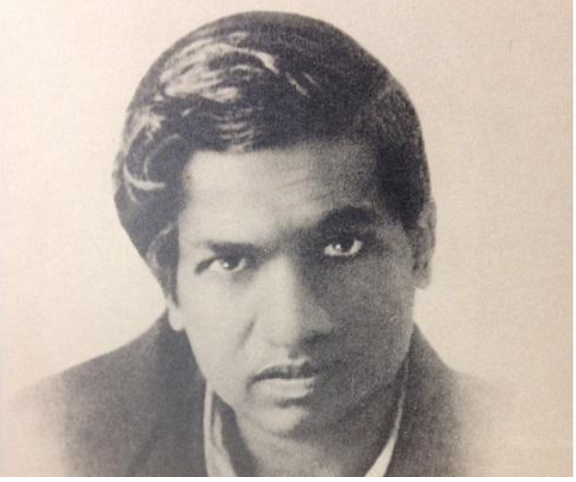
In this research thesis, (part XI) we analyze further equations concerning the Ramanujan Manuscript Book 1. We obtain new possible mathematical connections with various topics regarding the Dark Matter, the String Theory, in particular the Supersymmetry Breaking.

---

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni “R. Caccioppoli” - Università degli Studi di Napoli “Federico II” – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Vesuvius landscape with gorse – Naples

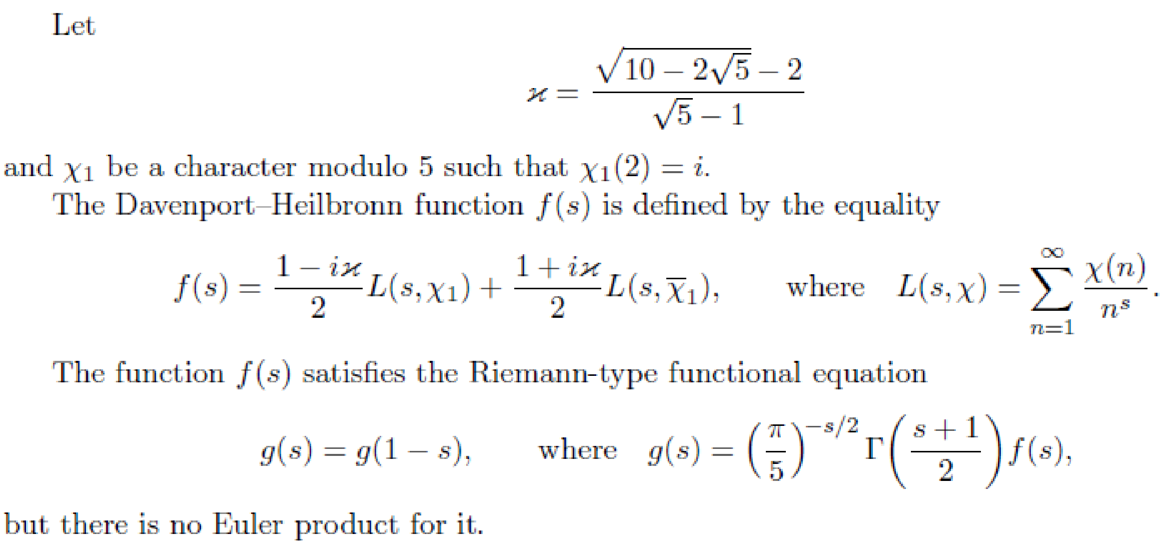
https://www.pinterest.it/pin/95068242114589901/

From:

On the Zeros of the Davenport Heilbronn Function
S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

A. A. Karatsuba, “On the zeros of arithmetic Dirichlet series without Euler product,” Izv. Ross. Akad. Nauk, Ser. Mat. 57 (5), 3–14 (1993)

We have:



Let

$$\varkappa = \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

and χ_1 be a character modulo 5 such that $\chi_1(2)=i$.

The Davenport–Heilbronn function $f(s)$ is defined by the equality

$$f(s) = \frac{1-i\varkappa}{2}L(s,\chi_1) + \frac{1+i\varkappa}{2}L(s,\overline{\chi}_1), \qquad \text{where} \quad L(s,\chi) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}.$$

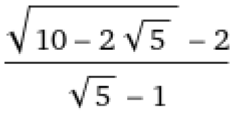
The function $f(s)$ satisfies the Riemann-type functional equation

$$g(s) = g(1-s), \qquad \text{where} \quad g(s) = \left(\frac{\pi}{5}\right)^{-s/2}\Gamma\left(\frac{s+1}{2}\right)f(s),$$

but there is no Euler product for it.

$$(\sqrt{10-2\sqrt{5}}-2)/(\sqrt{5}-1) = \kappa$$

Input:



$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation:

0.2840790438404122960282918323931261690910880884457375827591626661

…

0.28407904384…. = κ

Alternate forms:

$$\frac{1}{4}\left(\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2\right)$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

$$\frac{1}{2}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

Minimal polynomial:

$$x^4+2x^3-6x^2-2x+1$$

Expanded forms:

$$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}-\frac{2}{\sqrt{5}-1}$$

$$\frac{1}{4}\sqrt{10-2\sqrt{5}}+\frac{1}{4}\sqrt{5\left(10-2\sqrt{5}\right)}+\frac{1}{2}\left(-1-\sqrt{5}\right)$$

For (((($\sqrt{}$(10-2$\sqrt{}$5) -2))/((√5-1)))) = 8πG;   G = 0.011303146014

Indeed:

(((($\sqrt{}$(10-2$\sqrt{}$5) -2))/((√5-1))))/(8π)

Input:

$$\frac{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}{8\pi}$$

Result:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{8\left(\sqrt{5}-1\right)\pi}$$

Decimal approximation:

0.011303146014005214797375012944203574468576031392001780859490966 7...

0.01130314…. = g (gravitational coupling constant)

Property:

$$\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$$ is a transcendental number

Alternate forms:

$$\frac{\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

Expanded forms:

$$-\frac{1}{16\pi}-\frac{\sqrt{5}}{16\pi}+\frac{\sqrt{10-2\sqrt{5}}}{32\pi}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{32\pi}$$

$$\frac{\sqrt{10-2\sqrt{5}}}{8\left(\sqrt{5}-1\right)\pi}-\frac{1}{4\left(\sqrt{5}-1\right)\pi}$$

Series representations:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(9-2\sqrt{5}\right)^{-k}}{k!}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)} = \frac{-2+\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}}{8\pi\left(-1+\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(5-z_0\right)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We note that:

(((√(10-2√5) -2))/((√5-1)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))

Input:

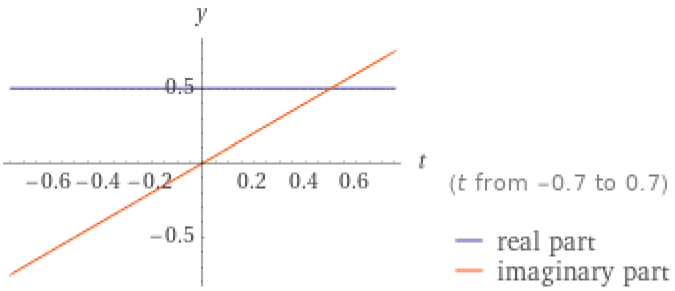
$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Plot:



Alternate form assuming t>0:

$$\frac{i\sqrt{10-2\sqrt{5}}\,t}{\sqrt{2(5-\sqrt{5})}-2}-\frac{2it}{\sqrt{2(5-\sqrt{5})}-2}+\frac{\sqrt{5(10-2\sqrt{5})}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}-\frac{\sqrt{5}}{(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}+\frac{1}{(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}$$

Alternate forms:

$$\frac{1}{8}(1+\sqrt{5})\left(2i\sqrt{2(3-\sqrt{5})}\,t+\sqrt{5}-1\right)$$

$$\frac{1}{2}(1+2it)$$

$$\frac{1}{2}+it$$

1/2+*it* = *real part of every nontrivial zero of the Riemann zeta function*

Derivative:

$$\frac{d}{dt}\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)(2i(\sqrt{5}-1)t+\sqrt{5}-1)}{(\sqrt{5}-1)\left(2\left(\sqrt{2(5-\sqrt{5})}-2\right)\right)}\right)=i$$

Indefinite integral:

$$\int \frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\,dt = \frac{t}{2}+\frac{i\,t^2}{2} + \text{constant}$$

And again:

(((√(10-2√5) -2)⁄((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2))) = (1/2+it)

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2\,x}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}=\frac{1}{2}+i\,t$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are real:

$$\frac{\sqrt{5}-1}{x}=2$$

Alternate form:

$$\frac{\left(\sqrt{5}-1\right)(1+2\,i\,t)}{4\,x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are positive:

$$2\,x+1=\sqrt{5}$$

Expanded forms:

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{10-2\sqrt{5}}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{5}\;t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+$$
$$\frac{i\,t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{10-2\sqrt{5}}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-$$
$$\frac{\sqrt{5}}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

$$\frac{i\sqrt{5}\;t}{2\,x}-\frac{i\,t}{2\,x}+\frac{\sqrt{5}}{4\,x}-\frac{1}{4\,x}=\frac{1}{2}+i\,t$$

Solutions:

$$t=\frac{i}{2}\,,\quad x\neq 0$$

$$x=\frac{\sqrt{5}}{2}-\frac{1}{2}$$

Input:

$$\frac{\sqrt{5}}{2}-\frac{1}{2}$$

Decimal approximation:

0.6180339887498948482045868343656381177203091798057628621354486227
…

$0.6180339887\ldots. = \frac{1}{\phi}$

Solution for the variable x:

$$x=\frac{-2i\sqrt{5}\,t+2it-\sqrt{5}+1}{-2-4it}$$

Implicit derivatives:

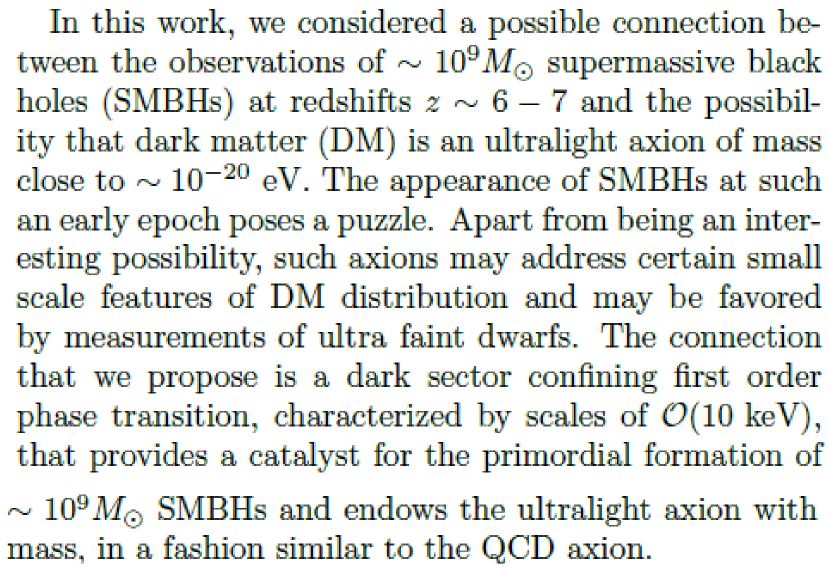
$$\frac{\partial x(t)}{\partial t}=\frac{2\left(-1+\sqrt{5}-2x\right)x}{\left(-1+\sqrt{5}\right)(-i+2t)}$$

$$\frac{\partial t(x)}{\partial x}=\frac{\left(-1+\sqrt{5}\right)(-i+2t)}{2\left(-1+\sqrt{5}-2x\right)x}$$

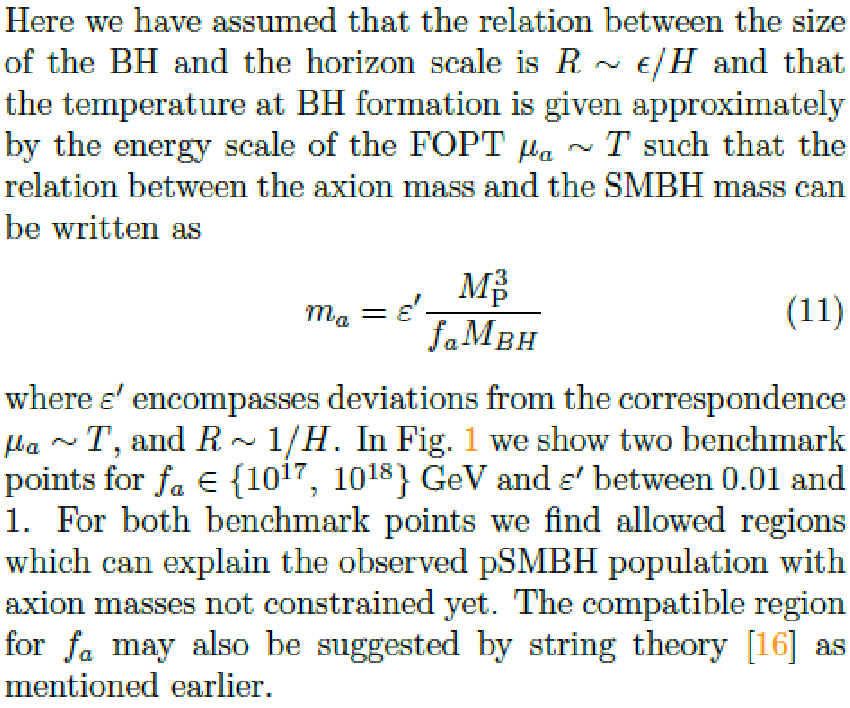
From:

Connecting the Extremes: A Story of Supermassive Black Holes and Ultralight Dark Matter - *Hooman Davoudiasl, Peter B. Denton and Julia Gehrlein* - arXiv:2109.01678v1 [astro-ph.CO] 3 Sep 2021

We have:



In this work, we considered a possible connection between the observations of $\sim 10^9 M_\odot$ supermassive black holes (SMBHs) at redshifts $z \sim 6-7$ and the possibility that dark matter (DM) is an ultralight axion of mass close to $\sim 10^{-20}$ eV. The appearance of SMBHs at such an early epoch poses a puzzle. Apart from being an interesting possibility, such axions may address certain small scale features of DM distribution and may be favored by measurements of ultra faint dwarfs. The connection that we propose is a dark sector confining first order phase transition, characterized by scales of $\mathcal{O}(10 \text{ keV})$, that provides a catalyst for the primordial formation of $\sim 10^9 M_\odot$ SMBHs and endows the ultralight axion with mass, in a fashion similar to the QCD axion.



Here we have assumed that the relation between the size of the BH and the horizon scale is $R \sim \epsilon/H$ and that the temperature at BH formation is given approximately by the energy scale of the FOPT $\mu_a \sim T$ such that the relation between the axion mass and the SMBH mass can be written as

$$m_a = \varepsilon' \frac{M_P^3}{f_a M_{BH}} \tag{11}$$

where ε' encompasses deviations from the correspondence $\mu_a \sim T$, and $R \sim 1/H$. In Fig. 1 we show two benchmark points for $f_a \in \{10^{17}, 10^{18}\}$ GeV and ε' between 0.01 and 1. For both benchmark points we find allowed regions which can explain the observed pSMBH population with axion masses not constrained yet. The compatible region for f_a may also be suggested by string theory [16] as mentioned earlier.

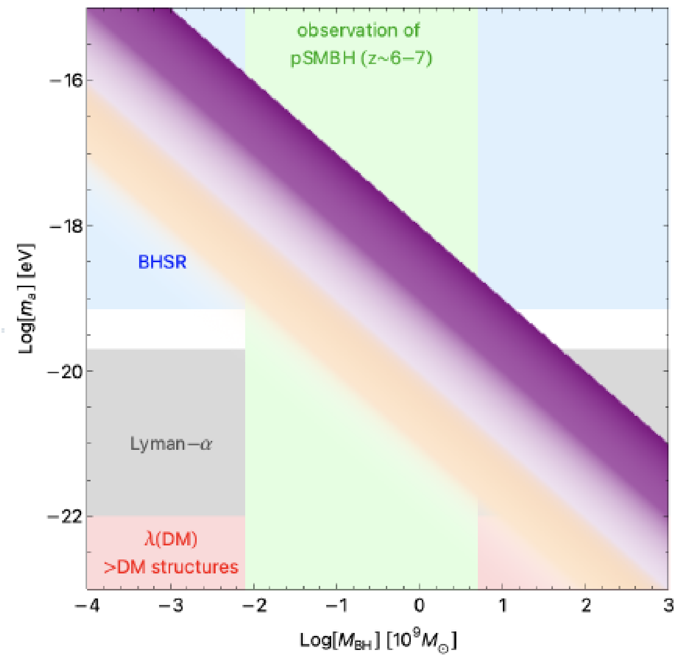


FIG. 1. Summary plot of the constraints and preferred regions of our model in the SMBH mass-axion mass plane. The green region shows the observed SMBH masses at redshift $\sim 6-7$. The blue region corresponds to constraints from BH superradiance (BHSR), the gray region shows the constraints from Lyman-α forest measurements, and in the red region the wavelength of DM exceeds the smallest observed DM structures which provides a lower bound on the DM mass (see main text for more details). The orange and purple regions provide two benchmark scenarios for the relation between the axion mass and the primordial SMBH mass given in Eq. (11) with $f_a = 10^{17}$ GeV (purple) and $f_a = 10^{18}$ GeV (orange). The color intensity represents a decrease in ε' from 1 to 0.01.

From eq.(11) :

(relation between the axion mass and the SMBH mass)

$$m_a = \varepsilon' \frac{M_{\mathrm{P}}^3}{f_a M_{BH}}$$

For f_a = 1*10^17 Gev = Kg

Input interpretation

convert 1×10^{17} GeV (gigaelectronvolts) to kilograms

Result

1.783×10^{-10} kg (kilograms)

(using $E = mc^2$)

We obtain:

((((((6.139608*10^-9)^3 / (1.783*10^-10*13.12806*10^39)))))) kg = eV

Input interpretation

$$\text{convert } \frac{(6.139608 \times 10^{-9})^3}{1.783 \times 10^{-10} \times 13.12806 \times 10^{39}} \text{ kg (kilograms)}$$

to electronvolts per speed of light squared

Result

5.546×10^{-20} eV/c^2

$5.546*10^{-20}$ eV = possible Axion mass

Dark Energy and Dark Matter

We consider $\Lambda = 1.1056 * 10^{-52}$ that is the value of the Cosmological Constant and 299792458 , that is the value of the speed of light in *m/s* , the terms of the Einstein equation $E = mc^2$ ($E = 1.1056 * 10^{-52}$ (Λ = Dark Energy)). Thence, we can to obtain the mass *m*:

(1.1056*10^-52)/(299792458)^2

Input interpretation

$$\frac{1.1056 \times 10^{-52}}{299\,792\,458^2}$$

Result

$1.230145901972880538611673865135959846721287859555184012098\ldots \times 10^{-69}$

$1.2301459019\ldots*10^{-69}$

From (**Ramanujan Manuscript Book 1** – *Srinivasa Ramanujan*):

From:

$$\frac{d}{d\beta}\left(\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times4}\right)^2\beta^2}\right)=\frac{450\left(\beta^2-\beta-8\right)}{\left(9\beta^2+16\beta+64\right)^2}$$

we obtain, for β = 8.7 and 37.6 :

(450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2

Input

$$\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times8.7+9\times8.7^2\right)^2}$$

Result

$0.0339377949190686866502717045312159360154663404711556830674890395\ldots$

0.033937794919….

(450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2

Input

$$\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times37.6+9\times37.6^2\right)^2}$$

Result

0.0034341920297328709820193684390072521962041927464073680700048781…

0.00343419202….

From the two previous analyzed expressions:

((((450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2 + (450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2)))^48

Input

$$\left(\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}+\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}\right)^{48}$$

Result

$3.0354078186632568216366107605966769350712011813380301580071\ldots \times 10^{-69}$

$3.035407818\ldots*10^{-69}$

And:

1/(([2.6 -25/9 ln(2.6(9*2.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*2.6+8)/(16sqrt2)))))]+[17.6 -25/9 ln(17.6(9*17.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*17.6+8)/(16sqrt2)))))]))^72

Input

$$1\Big/\left(\left(2.6-\frac{25}{9}\log(2.6\,(9\times 2.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)\right)\right)+\left(17.6-\frac{25}{9}\log(17.6\,(9\times 17.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)\right)\right)\right)^{72}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result

$1.24433\ldots \times 10^{-69}$

(result in radians)

$1.24433\ldots{*}10^{-69}$

we obtain, after some calculations:

(16 sqrt(2/3) log^(3/2)(2))/(e^(5/2) log^2(3)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69))

where

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^{2}(3)}$$

= 0.5127284721865…..

Input interpretation

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^{2}(3)}\left(\frac{1}{1+\zeta(2)^2}\left(3.035407\times 10^{-69}+1.24433\times 10^{-69}\right)+1.24433\times 10^{-69}\right)$$

$\log(x)$ is the natural logarithm

$\zeta(s)$ is the Riemann zeta function

Result

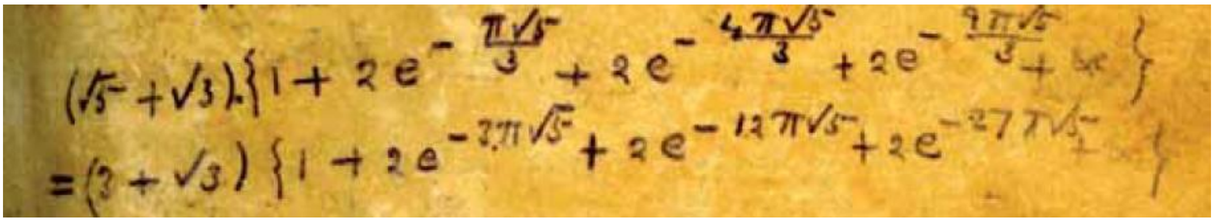
$1.23014\ldots \times 10^{-69}$

$1.23014\ldots{*}10^{-69}$

From:

Ramanujan Manuscript Book 1 – *Srinivasa Ramanujan*

From the following expressions



$$(\sqrt{5}+\sqrt{3})\left\{1+2e^{-\frac{\pi\sqrt{5}}{3}}+2e^{-\frac{4\pi\sqrt{5}}{3}}+2e^{-\frac{9\pi\sqrt{5}}{3}}+\&c\right\}$$
$$=(3+\sqrt{3})\left\{1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}+\&c\right\}$$

$$(3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)$$

$$(\sqrt{5}+\sqrt{3})\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times4(\pi\sqrt{5})}+2e^{-1/3\times9(\pi\sqrt{5})}\right)$$

after some calculations, we obtain:

((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)])))*2*0.94693

where 0.94693331865 = 0.937 + 0.9568666373 / 2

and 0.9568666373 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}}\approx 0.9568666373$$

Input

$$\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+(\sqrt{5}+\sqrt{3})\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times4(\pi\sqrt{5})}+2e^{-1/3\times9(\pi\sqrt{5})}\right)\right)\times2\times0.94693$$

Result

17.9237…

17.9237

Series representations

$$\Big((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\\ (\sqrt{5}+\sqrt{3})\left(1+2e^{1/3(\pi\sqrt{5})(-1)}+2e^{1/3(4(\pi\sqrt{5}))(-1)}+2e^{1/3(9(\pi\sqrt{5}))(-1)}\right)\Big)$$

$$2\times 0.94693=1.89386\,e^{-27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}$$

$$\Bigg(6+6e^{15\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+6e^{25\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+3e^{27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+\\ 2\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+2e^{15\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+\\ 2e^{25\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+e^{27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sqrt{2}\\ \sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+2e^{24\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)+\\ 2e^{77/3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)+\\ 2e^{80/3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)+\\ e^{27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)\Bigg)$$

$$\left(\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\right.$$

$$\left.\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)\right)$$

$$2\times 0.94693=1.89386\exp\left(-27\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left(6+6\exp\left(15\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+6\exp\left(25\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\right.$$

$$3\exp\left(27\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+2\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$

$$2\exp\left(15\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$

$$2\exp\left(25\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$

$$\exp\left(27\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$

$$2\exp\left(24\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}+$$

$$2\exp\left(\frac{77}{3}\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}+$$

$$2\exp\left(\frac{80}{3}\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}+$$

$$\left.\exp\left(27\,\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}\right)$$

$$\Big(\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+$$

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)\Big)$$

$$2\times 0.94693=1.89386\,e^{-27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}$$

$$\Bigg(6+6\,e^{15\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+6\,e^{25\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+$$

$$3\,e^{27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+2\,e^{24\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}$$

$$\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$2\,e^{\frac{77}{3}\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$2\,e^{\frac{80}{3}\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$e^{27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+$$

$$2\,e^{15\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+$$

$$2\,e^{25\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+$$

$$e^{27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\Bigg)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From the previous expression

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)$$

and the cosmological constant, after some calculations:

$$\frac{e^{-\frac{2}{3}+\frac{2}{3e}-\frac{2e}{3}+\frac{1}{\pi}+\pi}\,\pi^{2/3-(4e)/3}}{\sin^{5/3}(e\,\pi)}\times 1.1056\times 10^{-52}\times \frac{1}{\left(\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)\right)^{24}}$$

Result

$1.23041\ldots\times 10^{-69}$

$1.23041\ldots*10^{-69}$

From:

1/((1.23041*10^-69)*3.264346*10^94) (((((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)]+(sqrt5+sqrt3)[1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)])))*2*0.94693)))

where $1.23041 * 10^{-69}$ is the result that we have obtained previously and $3.264346 * 10^{94}$ is the value of the Planck Density. Multiplying the two values and the above expression, we obtain:

Input interpretation

$$\frac{1}{1.23041\times 10^{-69}\times 3.264346\times 10^{94}}\left(\left(\left(3+\sqrt{3}\right)\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4\pi\sqrt{5}}+2e^{-1/3\times 9\pi\sqrt{5}}\right)\right)\times 2\times 0.94693\right)$$

Result

$4.46253\ldots \times 10^{-25}$

$4.46253\ldots{*}10^{-25}$

From which, multiplying by 196884, we obtain:

196884(1/((1.23041e-69)*3.264346e+94))(((((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)]+(sqrt5+sqrt3)[1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)])))*2*0.94693)))

Where 196884 is is a fundamental number of the following *j*-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's *j*-invariant or *j* function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of *j* have to do with its *q* expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that *j* has a simple pole at the cusp, so its *q*-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

Furthermore, 196884 is the coefficient of q of the partition function $Z_1(q)$ that is the number of quantum states of the minimal black hole for the value of k equal to 1.

Input interpretation

$$
\begin{aligned}
196\,884 \times \frac{1}{1.23041\times 10^{-69}\times 3.264346\times 10^{94}} \\
\Big(\Big(\big(3+\sqrt{3}\big)\big(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\big)+\big(\sqrt{5}+\sqrt{3}\big) \\
\big(1+2\,e^{-(\pi\sqrt{5})/3}+2\,e^{-1/3\times 4(\pi\sqrt{5})}+2\,e^{-1/3\times 9(\pi\sqrt{5})}\big)\Big)\times 2\times 0.94693\Big)
\end{aligned}
$$

Result

$8.78602\ldots \times 10^{-20}$

$8.78602\ldots * 10^{-20}$ = possible Axion mass

From which:

-76/(ln(8.78602 × 10^-20)-Pi)

Input interpretation

$$-\frac{76}{\log\left(8.78602\times 10^{-20}\right)-\pi}$$

$\log(x)$ is the natural logarithm

Result

1.6163289137978991601368495324612473105715209116293674659533561228…

1.61632891379…… result that is a very good approximation to the value of the golden ratio 1.618033988749...

36[-(ln(8.78602 × 10^-20)-4-(((√(10-2√5) -2))/((√5-1))))]-2π+1/2

Input interpretation

$$36\left(-\left(\log(8.78602\times 10^{-20})-4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-2\pi+\frac{1}{2}$$

log(*x*) is the natural logarithm

Result

1728.0711…

1728.0711…….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

((36[-(ln(8.78602 × 10^-20)-4-(((√(10-2√5) -2))/((√5-1))))]-2π+1/2))^1/15

Input interpretation

$$\sqrt[15]{36\left(-\left(\log(8.78602\times 10^{-20})-4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-2\pi+\frac{1}{2}}$$

log(*x*) is the natural logarithm

Result

1.64375634…

1.64375634….. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27((36[-(ln(8.78602 × 10^-20)-4-(((√(10-2√5) -2))/((√5-1))))]-2π+1/2)))^2-1/3

Input interpretation

$$\left(\frac{1}{27}\left(36\left(-\left(\log(8.78602\times 10^{-20})-4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-2\pi+\frac{1}{2}\right)\right)^2-\frac{1}{3}$$

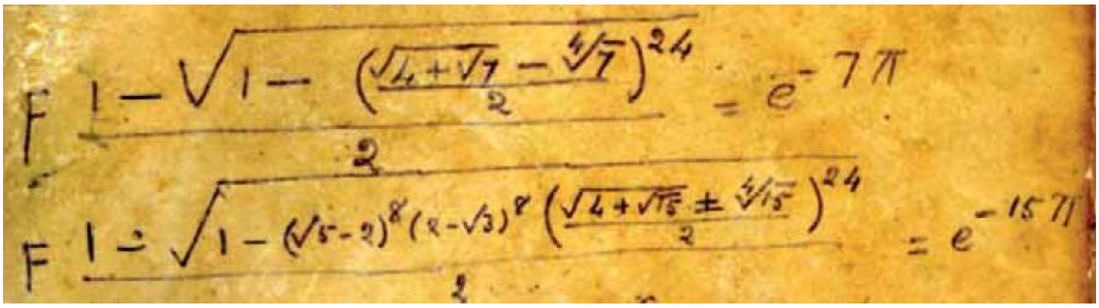
log(x) is the natural logarithm

Result

4096.004…

4096.004…. ≈ 4096 = 64^2

Now, we have that:



$$F\ \frac{1-\sqrt{1-\left(\frac{\sqrt{4+\sqrt{7}}-\sqrt[4]{7}}{2}\right)^{24}}}{2}=e^{-7\pi}$$

$$F\ \frac{1-\sqrt{1-(\sqrt{5}-2)^{8}(2-\sqrt{3})^{8}\left(\frac{\sqrt{4+\sqrt{15}}\pm\sqrt[4]{15}}{2}\right)^{24}}}{2}=e^{-15\pi}$$

From the first expression, we obtain:

((1-(1-(((4+sqrt7)^0.5-7^(1/4))/2)^24)^0.5)/2)

Input

$$\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{24}}\right)$$

Exact result

$$\frac{1}{2}\left(1-\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16777216}}\right)$$

Decimal approximation

$4.502829521839147472415970786519179155280869815850112562496\ldots\times 10^{-9}$

$4.50282952\ldots*10^{-9}$

Alternate forms

$$\frac{1}{2}\left(1-\sqrt{1-\frac{\left(\sqrt[4]{7}-\sqrt{4+\sqrt{7}}\right)^{24}}{16777216}}\right)$$

$$\frac{1}{2}-\sqrt{\left(6\left(-578340-218592\sqrt{7}+137923\sqrt[4]{7}\sqrt{4+\sqrt{7}}+52130\times 7^{3/4}\sqrt{4+\sqrt{7}}\right)\right)}$$

$$\frac{1}{2}\left(1-2\sqrt{\left(6\left(-578340-218592\sqrt{7}+137923\sqrt[4]{7}\sqrt{4+\sqrt{7}}+52130\times 7^{3/4}\sqrt{4+\sqrt{7}}\right)\right)}\right)$$

$$\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{\left(\sqrt[4]{7}-\sqrt{4+\sqrt{7}}\right)^{24}}{16777216}}$$

Minimal polynomial

$$256x^8-1024x^7+3553322752x^6-10659964672x^5+9426752608x^4-1086898624x^3-1011128720x^2-222082576x+1$$

Expanded form

$$\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16777216}}$$

e^(-7Pi)

Input

$e^{-7\pi}$

Decimal approximation

$2.8142684574855527210911764090117449751701270066564895149825\ldots \times 10^{-10}$

$2.814268457\ldots.*10^{-10}$

Property

$e^{-7\pi}$ is a transcendental number

Alternative representations

$$e^{-7\pi} = e^{-1260\,^{\circ}}$$

$$e^{-7\pi} = e^{7\,i\log(-1)}$$

$$e^{-7\pi} = \exp^{-7\pi}(z) \quad \text{for } z = 1$$

Series representations

$$e^{-7\pi} = e^{-28\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$e^{-7\pi} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-7\pi}$$

$$e^{-7\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-7\pi}$$

Integral representations

$$e^{-7\pi} = e^{-28\int_0^1 \sqrt{1-t^2}\, dt}$$

$$e^{-7\pi} = e^{-14\times\int_0^1 1/\sqrt{1-t^2}\, dt}$$

$$e^{-7\pi} = e^{-14\times\int_0^\infty 1/(1+t^2)\, dt}$$

We have the following equation:

((1-(1-((((4+sqrt7)^0.5-7^(1/4))/2)^24)^0.5)/2)*x = e^(-7Pi)

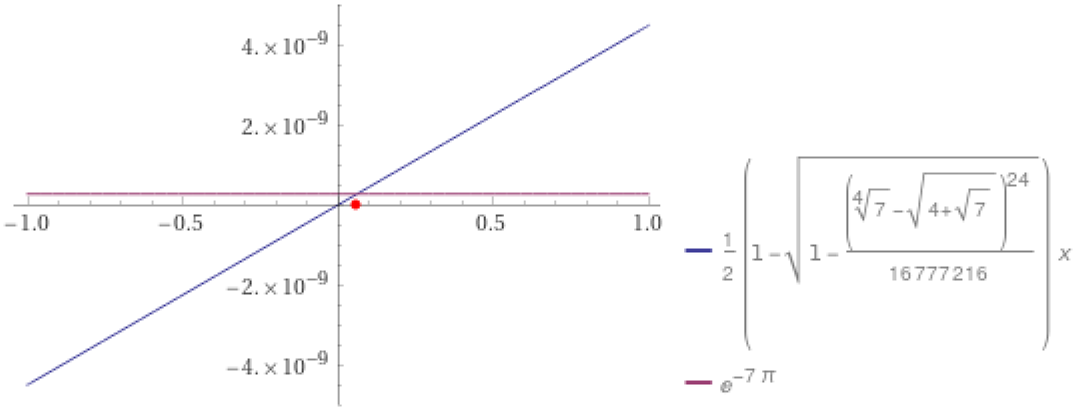
Input

$$\left(\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{24}}\right)\right)x = e^{-7\pi}$$

Exact result

$$\frac{1}{2}\left(1-\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16\,777\,216}}\right)x = e^{-7\pi}$$

Plot



Alternate forms

$$\frac{1}{2}\left(1-\sqrt{1-\frac{\left(\sqrt[4]{7}-\sqrt{4+\sqrt{7}}\right)^{24}}{16777216}}\right)x = e^{-7\pi}$$

$$-\frac{1}{2}\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16777216}}\;x+\frac{x}{2}-e^{-7\pi}=0$$

$$-\frac{1}{2}\left(2\sqrt{\left(6\left(-578340-218592\sqrt{7}+137923\sqrt[4]{7}\sqrt{4+\sqrt{7}}+52130\times 7^{3/4}\sqrt{4+\sqrt{7}}\right)\right)}-1\right)x = e^{-7\pi}$$

Solution

$x \approx 0.062500$

x = 0.0625000001407134 ≈ 1/16

Thence:

((1-(1-(((4+sqrt7)^0.5-7^(1/4))/2)^24)^0.5)/2)*1/16

Input

$$\left(\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{24}}\right)\right)\times\frac{1}{16}$$

Exact result

$$\frac{1}{32}\left(1-\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16777216}}\right)$$

Decimal approximation

$2.8142684511494671702599817415744869720505436349063203515601\ldots \times 10^{-10}$

$2.8142684511\ldots * 10^{-10}$

Alternate forms

$$\frac{1}{32}\left(1-\sqrt{1-\frac{\left(\sqrt[4]{7}-\sqrt{4+\sqrt{7}}\right)^{24}}{16777216}}\right)$$

$$\frac{1}{32}\left(1-2\sqrt{\left(6\left(-578340-218592\sqrt{7}+137923\sqrt[4]{7}\sqrt{4+\sqrt{7}}+52130\times 7^{3/4}\sqrt{4+\sqrt{7}}\right)\right)}\right)$$

$$\frac{1}{32} - \frac{1}{8} \sqrt{\left(\frac{3}{2} \left(-578\,340 - 218\,592 \sqrt{7} + 137\,923 \sqrt[4]{7} \sqrt{4 + \sqrt{7}} + 52\,130 \times 7^{3/4} \sqrt{4 + \sqrt{7}}\right)\right)}$$

$$\frac{1}{32} - \frac{1}{32} \sqrt{1 - \frac{\left(\sqrt[4]{7} - \sqrt{4 + \sqrt{7}}\right)^{24}}{16\,777\,216}}$$

Minimal polynomial

$$1\,099\,511\,627\,776\, x^8 - 274\,877\,906\,944\, x^7 + 59\,614\,863\,328\,018\,432\, x^6 - 11\,177\,783\,115\,907\,072\, x^5 + 617\,791\,658\,917\,888\, x^4 - 4\,451\,936\,763\,904\, x^3 - 258\,848\,952\,320\, x^2 - 3\,553\,321\,216\, x + 1$$

Expanded form

$$\frac{1}{32} - \frac{1}{32} \sqrt{1 - \frac{\left(\sqrt{4 + \sqrt{7}} - \sqrt[4]{7}\right)^{24}}{16\,777\,216}}$$

From the second expression, we obtain:

((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)

Input

$$\frac{1}{2} \left(1 - \sqrt{1 - \left(\sqrt{5} - 2\right)^8 \left(2 - \sqrt{3}\right)^8 \left(\frac{1}{2} \left(\sqrt{4 + \sqrt{15}} + \sqrt[4]{15}\right)\right)^{24}}\right)$$

Exact result

$$\frac{1}{2}\left(1-\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)$$

Decimal approximation

0.0816238866269037530355657224873764004700088138630573976803909208

…

0.0816238866269….

Minimal polynomial

$$\begin{aligned}&65\,536\,x^{16}-524\,288\,x^{15}+74\,797\,188\,356\,080\,563\,781\,632\,x^{14}-\\&523\,580\,318\,492\,563\,937\,296\,384\,x^{13}+1\,591\,941\,453\,578\,545\,744\,953\,344\,x^{12}-\\&2\,745\,104\,581\,067\,943\,308\,722\,176\,x^{11}+\\&2\,937\,189\,638\,441\,112\,810\,102\,784\,x^{10}-2\,001\,153\,789\,822\,190\,830\,313\,472\,x^9+\\&884\,084\,139\,026\,423\,472\,342\,528\,x^8-333\,568\,339\,080\,884\,549\,212\,160\,x^7+\\&205\,827\,032\,529\,626\,739\,642\,880\,x^6-120\,283\,099\,642\,602\,178\,798\,080\,x^5+\\&30\,013\,526\,524\,194\,687\,668\,672\,x^4-245\,664\,795\,505\,674\,391\,936\,x^3+\\&101\,075\,478\,019\,956\,932\,064\,x^2-18\,261\,032\,313\,496\,230\,944\,x+1\end{aligned}$$

Expanded form

$$\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}$$

Alternate forms

$$\frac{1}{2}-\frac{1}{2}\sqrt{1+\left(\frac{679\sqrt{3}}{1\,048\,576}-\frac{18\,817}{16\,777\,216}\right)\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}$$

$$\frac{1}{2}\left(1-\sqrt{1-\frac{\left(\sqrt{3}-2\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)$$

$$\frac{1}{2}\left(1-\sqrt{\left(4\ \boxed{\begin{array}{l}\text{root of }\ 65536x^8+74797188356080561946624x^7+\\ \quad 21200498100853939486720x^6+1280574465484047835136x^5+\\ \quad 29690604366209967998464x^4+\\ \quad 80035904092752989696x^3+82814445706460701120x^2+\\ \quad 18261032313496230944x+1\ \text{ near }\ x=-0.0749614\end{array}}+1\right)}\right)$$

$$\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{\left(\sqrt{3}-2\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}$$

e^(-15Pi)

Input

$e^{-15\pi}$

Decimal approximation

$3.4225885441212408677962004403087394257136035137816749401852\ldots\times 10^{-21}$

$3.42258854412\ldots*10^{-21}$

Property

$e^{-15\pi}$ is a transcendental number

Alternative representations

$$e^{-15\pi} = e^{-2700^\circ}$$

$$e^{-15\pi} = e^{15 i \log(-1)}$$

$$e^{-15\pi} = \exp^{-15\pi}(z) \quad \text{for } z = 1$$

Series representations

$$e^{-15\pi} = e^{-60 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{-15\pi} = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-15\pi}$$

$$e^{-15\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-15\pi}$$

Integral representations

$$e^{-15\pi} = e^{-60 \int_0^1 \sqrt{1-t^2}\, dt}$$

$$e^{-15\pi} = e^{-30\times\int_0^1 1/\sqrt{1-t^2}\,dt}$$

$$e^{-15\pi} = e^{-30\times\int_0^\infty 1/(1+t^2)\,dt}$$

Dividing the two sides, we obtain:

e^(-15Pi) / ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)

Input

$$\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^8\left(2-\sqrt{3}\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)}$$

Exact result

$$\frac{2\,e^{-15\pi}}{1-\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}}$$

Decimal approximation

$4.193121260894643406116986001114129213160283865355762814 1784\ldots\times 10^{-20}$

$4.19312126\ldots*10^{-20}$ = possible Axion mass

Property

$$\frac{2\,e^{-15\pi}}{1-\sqrt{1-\frac{(2-\sqrt{3})^8\,(-2+\sqrt{5})^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}} \text{ is a transcendental number}$$

Alternate forms

$$\frac{2\,e^{-15\pi}}{1-\sqrt{1+\left(\frac{679\sqrt{3}}{1\,048\,576}-\frac{18\,817}{16\,777\,216}\right)\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}}$$

$$-\left(\left(2\,e^{-15\pi}\right)\middle/\left(2\sqrt{\left(2\left(-71\,332\,157\,474\,594\,652+41\,183\,640\,319\,833\,998\sqrt{3}+\right.\right.}\right.\right.$$
$$31\,900\,710\,618\,982\,674\sqrt{5}-18\,417\,883\,863\,210\,000\sqrt{15}+$$
$$7\,458\,169\,532\,729\,328\times 3^{3/4}\sqrt[4]{5}\sqrt{4+\sqrt{15}}+$$
$$5\,777\,073\,278\,690\,448\sqrt[4]{3}\;5^{3/4}\sqrt{4+\sqrt{15}}-$$
$$12\,917\,928\,562\,149\,429\sqrt[4]{15}\sqrt{4+\sqrt{15}}-$$
$$\left.\left.\left.\left.3\,335\,394\,812\,580\,124\times 15^{3/4}\sqrt{4+\sqrt{15}}\right)\right)-1\right)\right)$$

$e^{-15\pi}$

root of $x^{16}-18\,261\,032\,313\,496\,230\,944\,x^{15}+$
$101\,075\,478\,019\,956\,932\,064\,x^{14}-245\,664\,795\,505\,674\,391\,936\,x^{13}+$
$30\,013\,526\,524\,194\,687\,668\,672\,x^{12}-120\,283\,099\,642\,602\,178\,798\,080\,x^{11}+$
$205\,827\,032\,529\,626\,739\,642\,880\,x^{10}-333\,568\,339\,080\,884\,549\,212\,160\,x^{9}+$
$884\,084\,139\,026\,423\,472\,342\,528\,x^{8}-2\,001\,153\,789\,822\,190\,830\,313\,472\,x^{7}+$
$2\,937\,189\,638\,441\,112\,810\,102\,784\,x^{6}-2\,745\,104\,581\,067\,943\,308\,722\,176\,x^{5}+$
$1\,591\,941\,453\,578\,545\,744\,953\,344\,x^{4}-523\,580\,318\,492\,563\,937\,296\,384\,x^{3}+$
$74\,797\,188\,356\,080\,563\,781\,632\,x^{2}-524\,288\,x+65\,536$ near $x=12.2513$

Series representations

$$\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^8\left(2-\sqrt{3}\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)} = \frac{2\,e^{-15\pi}}{1-\sqrt{1-\frac{\left(2-\sqrt{2}\,\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^8\left(-2+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{14}\,\sum_{k=0}^{\infty}14^{-k}\binom{\frac{1}{2}}{k}}\right)^{24}}{16\,777\,216}}}$$

$$\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^8\left(2-\sqrt{3}\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)} = \left(2\,e^{-15\pi}\right)\Bigg/\left(1-\sqrt{\left(1-\frac{1}{16\,777\,216}\left(2-\sqrt{2}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8\left(-2+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{14}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{14}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)^{24}\right)}\right)$$

$$\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-(\sqrt{5}-2)^8\,(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)}=$$

$$\left(2\,e^{-15\pi}\right)\Big/\left(1-\sqrt{\left(1-\frac{1}{16777216}\left(2-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)^8\left(-2+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(15-z_0)^k\,z_0^{-k}}{k!}}\right)^{24}\right)}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which, after some calculations:

[-(ln((e^(-15Pi) / ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)))+43)]

where 43 is a twin prime number

Input

$$-\left(\log\left(\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-(\sqrt{5}-2)^8\,(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)}\right)+43\right)$$

$\log(x)$ is the natural logarithm

Exact result

$$-43-\log\left(\frac{2e^{-15\pi}}{1-\sqrt{1-\frac{(2-\sqrt{3})^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}}\right)$$

Decimal approximation

1.6182564722691413349609313668855858675685796633529832234236854228

…

1.61825647226…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$-43-\log\left(\frac{2e^{-15\pi}}{1-\sqrt{1-\frac{(\sqrt{3}-2)^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}}\right)$$

$$-\log\left(e^{-15\pi}\ \text{root of}\ x^{16}-18\,261\,032\,313\,496\,230\,944\,x^{15}+101\,075\,478\,019\,956\,932\,064\,x^{14}-245\,664\,795\,505\,674\,391\,936\,x^{13}+30\,013\,526\,524\,194\,687\,668\,672\,x^{12}-120\,283\,099\,642\,602\,178\,798\,080\,x^{11}+205\,827\,032\,529\,626\,739\,642\,880\,x^{10}-333\,568\,339\,080\,884\,549\,212\,160\,x^{9}+884\,084\,139\,026\,423\,472\,342\,528\,x^{8}-2\,001\,153\,789\,822\,190\,830\,313\,472\,x^{7}+2\,937\,189\,638\,441\,112\,810\,102\,784\,x^{6}-2\,745\,104\,581\,067\,943\,308\,722\,176\,x^{5}+1\,591\,941\,453\,578\,545\,744\,953\,344\,x^{4}-523\,580\,318\,492\,563\,937\,296\,384\,x^{3}+74\,797\,188\,356\,080\,563\,781\,632\,x^{2}-524\,288\,x+65\,536\ \text{near}\ x=12.2513\right)-43$$

$$-43+15\pi-\log(2)+\log\left(1-\sqrt{1-\frac{(\sqrt{3}-2)^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)$$

$$-43+15\pi-\log(2)+\log\left(1-\sqrt{1-\frac{(2-\sqrt{3})^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)$$

Alternative representations

$$-\left(\log\left(\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)}\right)+43\right)=$$

$$-43-\log_e\left(\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)\right)^{24}(2-\sqrt{3})^8(-2+\sqrt{5})^8}\right)}\right)$$

$$-\left(\log\left(\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^{8}\left(2-\sqrt{3}\right)^{8}\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)}\right)+43\right)=$$

$$-43-\log(a)\log_{a}\left(\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)\right)^{24}\left(2-\sqrt{3}\right)^{8}\left(-2+\sqrt{5}\right)^{8}}\right)}\right)$$

$$-\left(\log\left(\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^{8}\left(2-\sqrt{3}\right)^{8}\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)}\right)+43\right)=$$

$$-43+\mathrm{Li}_{1}\left(1-\frac{e^{-15\pi}}{\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)\right)^{24}\left(2-\sqrt{3}\right)^{8}\left(-2+\sqrt{5}\right)^{8}}\right)}\right)$$

24* 4.1931212608946434 × 10^-20 ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)

Input interpretation

$$24\times 4.1931212608946434\times 10^{-20}\left(\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^8\left(2-\sqrt{3}\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)\right)$$

Result

$8.2142125058909781\ldots\times 10^{-20}$

$8.214212505\ldots*10^{-20}$ = possible Axion mass

Or:

24/(244829128850282463 π^4) ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)

Input

$$\frac{24}{244829128850282463\,\pi^4}\left(\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^8\left(2-\sqrt{3}\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)\right)$$

Exact result

$$\frac{4\left(1-\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)}{81609709616760821\,\pi^4}$$

Decimal approximation

$8.2142125058909780871400007310545544064197958454938059843015\ldots \times 10^{-20}$

8.21421250589…$*10^{-20}$ as above

Property

$$\frac{4\left(1-\sqrt{1-\frac{(2-\sqrt{3})^8(-2+\sqrt{5})^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)}{81609709616760821\,\pi^4} \text{ is a transcendental number}$$

From which:

-36(ln(24/(244829128850282463 π^4) ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2))-4)+3

Input

$$-36\left(\log\left(\frac{24}{244829128850282463\,\pi^4}\left(\frac{1}{2}\left(1-\sqrt{1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)\right)\right)-4\right)+3$$

log(x) is the natural logarithm

Exact result

$$3-36\left(\log\left(\frac{4\left(1-\sqrt{1-\frac{(2-\sqrt{3})^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)}{81609709616760821\,\pi^4}\right)-4\right)$$

Decimal approximation

1729.0500950459623064431262579259218388913808181125777234038961668

…

1729.0500950459623….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$147 - 36 \log\left(\frac{4\left(1 - \sqrt{1 - \frac{(\sqrt{3}-2)^8 (\sqrt{5}-2)^8 \left(\sqrt[4]{15} + \sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)}{81\,609\,709\,616\,760\,821\,\pi^4}\right)$$

$$-3\left(12 \log\left(\frac{4\left(1 - \sqrt{1 - \frac{(2-\sqrt{3})^8 (\sqrt{5}-2)^8 \left(\sqrt[4]{15} + \sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)}{81\,609\,709\,616\,760\,821\,\pi^4}\right) - 49\right)$$

$$3-36\left(-4-\log\left(\frac{81\,609\,709\,616\,760\,821}{4}\right)+\log\left(1-\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)-4\log(\pi)\right)$$

$$3-36\left(-4+\log(4)-\log(19)-\log(229)-\log(18\,756\,540\,936\,971)+\log\left(1-\sqrt{1-\frac{\left(\sqrt{3}-2\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)-4\log(\pi)\right)$$

$$3-36\left(-4+2\log(2)-\log(19)-\log(229)-\log(18\,756\,540\,936\,971)+\log\left(1-\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)-4\log(\pi)\right)$$

(-36(ln(24/(244829128850282463 π^4) ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2))-4)+3)^1/15

Input

$$\left(-36\left(\log\left(\frac{24}{244829128850282463\,\pi^4}\left(\frac{1}{2}\left(1-\sqrt{1-\left(\sqrt{5}-2\right)^8\left(2-\sqrt{3}\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)\right)\right)-4\right)+3\right)\hat{}\,(1/15)$$

log(*x*) is the natural logarithm

Exact result

$$\sqrt[15]{3-36\left(\log\left(\frac{4\left(1-\sqrt{1-\frac{\left(2-\sqrt{3}\right)^8\left(\sqrt{5}-2\right)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)}{81609709616760821\,\pi^4}\right)-4\right)}$$

Decimal approximation

1.6438184038359095639177817179925530631085421535605069940477367763
...

1.6438184038359….. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

Alternate forms

$$\sqrt[15]{147-36\log\left(\frac{4\left(1-\sqrt{1-\frac{(\sqrt{3}-2)^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)}{81609709616760821\,\pi^4}\right)}$$

$$\left(3-36\left(-4-\log\left(\frac{81609709616760821}{4}\right)+\log\left(1-\sqrt{1-\frac{(2-\sqrt{3})^8(\sqrt{5}-2)^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)-4\log(\pi)\right)\right)\text{ ^ }(1/15)$$

(1/27(-36(ln(24/(244829128850282463 π^4) ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2))-4)+2))^2-0.284079

Input interpretation

$$\left(\frac{1}{27}\left(-36\left(\log\left(\frac{24}{244829128850282463\,\pi^4}\left(\frac{1}{2}\left(1-\sqrt{\left(1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}\right)}\right)\right)\right)-4\right)+2\right)\right)^2-0.284079$$

$\log(x)$ is the natural logarithm

Result

4095.953412…

4095.953412…. ≈ 4096 = 64^2

Alternative representations

$$\left(\frac{1}{27}\left(-36\left(\log\left(\left(24\left(1-\sqrt{\left(1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}\right)}\right)\right)\Big/\left(\left(244829128850282463\,\pi^4\right)2\right)\right)-4\right)+2\right)\right)^2-0.284079=-0.284079+\left(\frac{1}{27}\left(2-36\left(-4+\log_e\left(\left(12\left(1-\sqrt{\left(1-\left(\frac{1}{2}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)\right)^{24}(2-\sqrt{3})^8(-2+\sqrt{5})^8\right)}\right)\right)\Big/\left(244829128850282463\,\pi^4\right)\right)\right)\right)\right)^2$$

$$\left(\frac{1}{27}\left(-36\left(\log\left(\left(24\left(1-\sqrt{\left(1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}\right)}\right)\right)\Big/\left(\left(244829128850282463\,\pi^4\right)2\right)\right)-4\right)+2\right)\right)^2-0.284079=-0.284079+\left(\frac{1}{27}\left(2-36\left(-4+\log(a)\log_a\left(\left(12\left(1-\sqrt{\left(1-\left(\frac{1}{2}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)\right)^{24}(2-\sqrt{3})^8(-2+\sqrt{5})^8\right)}\right)\right)\Big/\left(244829128850282463\,\pi^4\right)\right)\right)\right)\right)^2$$

$$\left(\frac{1}{27}\left(-36\left(\log\left(\left(24\left(1-\sqrt{\left(1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}\right)}\right)\right)\Big/\left(\left(244\,829\,128\,850\,282\,463\,\pi^4\right)2\right)\right)-4\right)+2\right)\right)^2-0.284079=-0.284079+\left(\frac{1}{27}\left(2-36\left(-4-\text{Li}_1\left(1-\left(12\left(1-\sqrt{\left(1-\left(\frac{1}{2}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)\right)^{24}(2-\sqrt{3})^8(-2+\sqrt{5})^8\right)}\right)\right)\Big/\left(244\,829\,128\,850\,282\,463\,\pi^4\right)\right)\right)\right)\right)^2$$

Integral representation

$$\left(\frac{1}{27}\left(-36\left(\log\left(\left(24\left(1-\sqrt{\left(1-(\sqrt{5}-2)^8(2-\sqrt{3})^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}\right)}\right)\right)\Big/\left(\left(244\,829\,128\,850\,282\,463\,\pi^4\right)2\right)\right)-4\right)+2\right)\right)^2-0.284079=-0.284079+\frac{1}{729}\left(2-36\left(-4+\int_1^{\frac{4\left(1-\sqrt{1-\frac{(2-\sqrt{3})^8(-2+\sqrt{5})^8\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)}{81\,609\,709\,616\,760\,821\,\pi^4}}\frac{1}{t}\,dt\right)\right)^2$$

From the two expressions, after some calculations, we obtain also:

(((((((1-(1-(((4+sqrt7)^0.5-7^(1/4))/2)^24)^0.5)/2)*1/16))) * 24/(244829128850282463 π^4) ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)))*1729*728^2

Input

$$\left(\left(\left(\left(\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{24}}\right)\right)\times\frac{1}{16}\right)\times\frac{24}{244\,829\,128\,850\,282\,463\,\pi^{4}}\left(\frac{1}{2}\left(1-\sqrt{1-(\sqrt{5}-2)^{8}\,(2-\sqrt{3})^{8}\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)\right)\right)\right)\times 1729\times 728^{2}$$

Exact result

$$\left(6\,028\,568\left(1-\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16\,777\,216}}\right)\left(1-\sqrt{1-\frac{(2-\sqrt{3})^{8}\,(\sqrt{5}-2)^{8}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16\,777\,216}}\right)\right)\Big/\left(4\,295\,247\,874\,566\,359\,\pi^{4}\right)$$

Decimal approximation

$2.1183084962437686963057730084125807025686843334422608871012\ldots\times 10^{-20}$

2.1183084962437…*10^{-20} = possible Axion mass

Property

$$\left(6028568\left(1-\sqrt{1-\frac{\left(-\sqrt[4]{7}+\sqrt{4+\sqrt{7}}\right)^{24}}{16777216}}\right)\left(1-\sqrt{1-\frac{(2-\sqrt{3})^{8}(-2+\sqrt{5})^{8}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)\right)\Big/\left(4295247874566359\,\pi^{4}\right)$$ is a transcendental number

Or, multiplying by 196884*1729π , we obtain:

(((((((1-(1-(((4+sqrt7)^0.5-7^(1/4))/2)^24)^0.5)/2)*1/16))) * 24/(244829128850282463 π^4) ((1-(1-(sqrt5-2)^8(2-sqrt3)^8(((4+sqrt15)^0.5+15^(1/4))/2)^24)^0.5)/2)))*196884*1729*Pi^2

Input

$$\left(\left(\left(\left(\frac{1}{2}\left(1-\sqrt{1-\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{24}}\right)\right)\times\frac{1}{16}\right)\times\frac{24}{244829128850282463\,\pi^{4}}\left(\frac{1}{2}\left(1-\sqrt{1-(\sqrt{5}-2)^{8}(2-\sqrt{3})^{8}\left(\frac{1}{2}\left(\sqrt{4+\sqrt{15}}+\sqrt[4]{15}\right)\right)^{24}}\right)\right)\right)\right)\times 196884\times 1729\,\pi^{2}$$

Exact result

$$\left(4479111\left(1-\sqrt{1-\frac{\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}}{16777216}}\right)\left(1-\sqrt{1-\frac{(2-\sqrt{3})^{8}(\sqrt{5}-2)^{8}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)\right)\Big/\left(8590495749132718\,\pi^{2}\right)$$

Decimal approximation

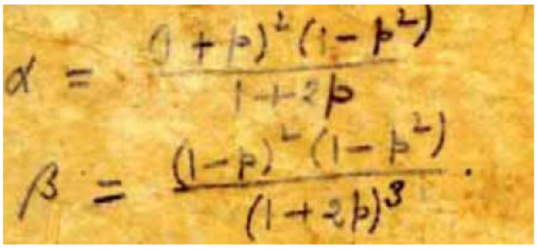
$7.7667015878797494895648377457513336867708307274102737396488\ldots \times 10^{-20}$

$7.76670158787\ldots{*}10^{-20}$ = possible Axion mass

Property

$$\left(4479111\left(1-\sqrt{1-\frac{\left(-\sqrt[4]{7}+\sqrt{4+\sqrt{7}}\right)^{24}}{16777216}}\right)\left(1-\sqrt{1-\frac{(2-\sqrt{3})^{8}(-2+\sqrt{5})^{8}\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)^{24}}{16777216}}\right)\right)\Big/\left(8590495749132718\,\pi^{2}\right)$$ is a transcendental number

From:



$$\alpha = \frac{(1+p)^2(1-p^2)}{1+2p}$$

$$\beta = \frac{(1-p)^2(1-p^2)}{(1+2p)^3}.$$

For p = 3:

(((1+3)^2(1-3^2))) / (1+2*3)

Input

$$\frac{(1+3)^2\left(1-3^2\right)}{1+2\times 3}$$

Exact result

$$-\frac{128}{7}$$

Decimal approximation

$-18.2857142857142857142857142857142857142857142857142857142857142857142 8\ldots$

-18.285714….

α = -128/7

(((1-3)^2(1-3^2))) / (1+2*3)^3

Input

$$\frac{(1-3)^2\left(1-3^2\right)}{(1+2\times 3)^3}$$

Exact result

$$-\frac{32}{343}$$

Decimal approximation

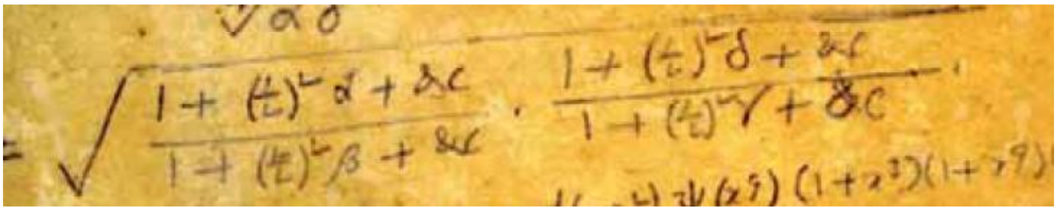
$-0.0932944606413994169096209912536443148688046647230320699708454 81$

…

-0.09329446……

β = -32/343

From:



$$= \sqrt{\frac{1+\left(\frac{1}{2}\right)^2\alpha+\&c}{1+\left(\frac{1}{2}\right)^2\beta+\&c} \cdot \frac{1+\left(\frac{1}{2}\right)^2\delta+\&c}{1+\left(\frac{1}{2}\right)^2\gamma+\&c}}$$

sqrt[((1+(1/2)^2*α)) / ((1+(1/2)^2*β)) * ((1+(1/2)^2*δ)) / ((1+(1/2)^2*γ))]

Input

$$\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\alpha}{1+\left(\frac{1}{2}\right)^2\beta} \times \frac{1+\left(\frac{1}{2}\right)^2\delta}{1+\left(\frac{1}{2}\right)^2\gamma}}$$

Result

$$\sqrt{\frac{\left(\frac{\alpha}{4}+1\right)\left(\frac{\delta}{4}+1\right)}{\left(\frac{\beta}{4}+1\right)\left(\frac{\gamma}{4}+1\right)}}$$

Alternate form

$$\sqrt{\frac{(\alpha+4)\,(\delta+4)}{(\beta+4)\,(\gamma+4)}}$$

Roots

$$\beta\,\gamma+4\,\beta+4\,\gamma+16\neq 0\,,\quad \alpha=-4$$

$$\alpha+4\neq 0\,,\quad \beta\,\gamma+4\,\beta+4\,\gamma+16\neq 0\,,\quad \delta=-4$$

Series expansion at α=0

$$2\sqrt{\frac{\delta+4}{(\beta+4)\,(\gamma+4)}}+\frac{1}{4}\,\alpha\sqrt{\frac{\delta+4}{(\beta+4)\,(\gamma+4)}}-\frac{1}{64}\,\alpha^2\sqrt{\frac{\delta+4}{(\beta+4)\,(\gamma+4)}}+\frac{1}{512}\,\alpha^3\sqrt{\frac{\delta+4}{(\beta+4)\,(\gamma+4)}}-\frac{5\,\alpha^4\sqrt{\frac{\delta+4}{(\beta+4)\,(\gamma+4)}}}{16\,384}+O\left(\alpha^5\right)$$

(Taylor series)

Series expansion at α=∞

$$\frac{\sqrt{\alpha}\sqrt{\frac{\alpha\,(\delta+4)}{(\beta+4)\,(\gamma+4)}}}{\sqrt{\alpha}}+\frac{2\sqrt{\frac{1}{\alpha}}\sqrt{\frac{\alpha\,(\delta+4)}{(\beta+4)\,(\gamma+4)}}}{\sqrt{\alpha}}-\frac{2\left(\frac{1}{\alpha}\right)^{3/2}\sqrt{\frac{\alpha\,(\delta+4)}{(\beta+4)\,(\gamma+4)}}}{\sqrt{\alpha}}+\frac{4\left(\frac{1}{\alpha}\right)^{5/2}\sqrt{\frac{\alpha\,(\delta+4)}{(\beta+4)\,(\gamma+4)}}}{\sqrt{\alpha}}+O\left(\left(\frac{1}{\alpha}\right)^3\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(\sqrt{\frac{\left(1+\left(\frac{1}{2}\right)^2\alpha\right)\left(1+\left(\frac{1}{2}\right)^2\delta\right)}{\left(1+\left(\frac{1}{2}\right)^2\beta\right)\left(1+\left(\frac{1}{2}\right)^2\gamma\right)}}\right)=\frac{\sqrt{\frac{(\alpha+4)\,(\delta+4)}{(\beta+4)\,(\gamma+4)}}}{2\,\alpha+8}$$

Indefinite integral

$$\int \sqrt{\frac{\left(1+\frac{\alpha}{4}\right)\left(1+\frac{\delta}{4}\right)}{\left(1+\frac{\beta}{4}\right)\left(1+\frac{\gamma}{4}\right)}}\, d\alpha = \frac{2}{3}(\alpha+4)\sqrt{\frac{(\alpha+4)(\delta+4)}{(\beta+4)(\gamma+4)}} + \text{constant}$$

sqrt[((1+(1/2)^2*(-128/7)) / ((1+(1/2)^2*(-32/343))) * ((1+(1/2)^2*δ)) / ((1+(1/2)^2*γ))]

Input

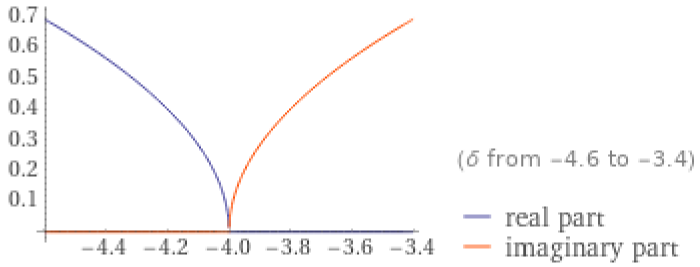
$$\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\left(-\frac{128}{7}\right)}{1+\left(\frac{1}{2}\right)^2\left(-\frac{32}{343}\right)} \times \frac{1+\left(\frac{1}{2}\right)^2\delta}{1+\left(\frac{1}{2}\right)^2\gamma}}$$

γ is the Euler-Mascheroni constant

Exact result

$$7\sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}}\sqrt{-\frac{\delta}{4}-1}$$

Plot



(δ from −4.6 to −3.4)

Alternate form

$$7\sqrt{\frac{5}{67(4+\gamma)}}\sqrt{-\delta-4}$$

Root

$\delta = -4$

Series expansion at δ=0

$$\begin{cases} -14\, i \sqrt{\frac{5}{67\,(4+\gamma)}} - \frac{7}{4}\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta + \frac{7}{64}\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^2 - \\ \quad \frac{7}{512}\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^3 + \frac{35\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^4}{16\,384} - \frac{49\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^5}{131\,072} + O\big(\delta^6\big) & \operatorname{Im}(\delta) > 0 \\ 14\, i \sqrt{\frac{5}{67\,(4+\gamma)}} + \frac{7}{4}\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta - \frac{7}{64}\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^2 + \\ \quad \frac{7}{512}\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^3 - \frac{35\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^4}{16\,384} + \frac{49\, i \sqrt{\frac{5}{67\,(4+\gamma)}}\, \delta^5}{131\,072} + O\big(\delta^6\big) & \text{(otherwise)} \end{cases}$$

Series expansion at δ=∞

$$\frac{7 \sqrt{\frac{5}{67\,(4+\gamma)}}\, \sqrt{-\delta}\, \sqrt{\delta}}{\sqrt{\delta}} + \frac{14 \sqrt{\frac{5}{67\,(4+\gamma)}}\, \sqrt{-\delta}\, \sqrt{\frac{1}{\delta}}}{\sqrt{\delta}} + \frac{14 \sqrt{\frac{5}{67\,(4+\gamma)}}\, \sqrt{\delta}\, \left(\frac{1}{\delta}\right)^{3/2}}{\sqrt{-\delta}} + \frac{28 \sqrt{\frac{5}{67\,(4+\gamma)}}\, \sqrt{-\delta}\, \left(\frac{1}{\delta}\right)^{5/2}}{\sqrt{\delta}} + O\left(\left(\frac{1}{\delta}\right)^3\right)$$

(generalized Puiseux series)

Derivative

$$\frac{d}{d\delta}\left(\sqrt{\frac{\left(1 + \frac{1}{7}\left(\frac{1}{2}\right)^2 \times (-128)\right)\left(1 + \left(\frac{1}{2}\right)^2 \delta\right)}{\left(1 + \frac{1}{343}\left(\frac{1}{2}\right)^2 \times (-32)\right)\left(1 + \left(\frac{1}{2}\right)^2 \gamma\right)}}\right) = -\frac{7 \sqrt{\frac{5}{67\,(4+\gamma)}}}{2 \sqrt{-\delta - 4}}$$

Indefinite integral

$$\int 7 \sqrt{\frac{5}{67\left(1 + \frac{\gamma}{4}\right)}}\, \sqrt{-1 - \frac{\delta}{4}}\, d\delta = -\frac{14}{3} \sqrt{\frac{5}{67\,(4+\gamma)}}\, (-\delta - 4)^{3/2} + \text{constant}$$

Global minimum

$$\min\left\{\sqrt{\frac{\left(1+\frac{1}{7}\left(\frac{1}{2}\right)^2(-128)\right)\left(1+\left(\frac{1}{2}\right)^2\delta\right)}{\left(1+\frac{1}{343}\left(\frac{1}{2}\right)^2(-32)\right)\left(1+\left(\frac{1}{2}\right)^2\gamma\right)}}\right\}=0 \text{ at } \delta=-4$$

Series representations

$$\sqrt{\frac{\left(1+\left(\frac{1}{2}\right)^2\delta\right)\left(1+\frac{1}{7}\left(\frac{1}{2}\right)^2(-128)\right)}{\left(1+\left(\frac{1}{2}\right)^2\gamma\right)\left(1+\frac{1}{343}\left(\frac{1}{2}\right)^2(-32)\right)}}=\sqrt{-\frac{1248+67\gamma+245\delta}{67(4+\gamma)}}\sum_{k=0}^{\infty}67^k\left(-\frac{1248+67\gamma+245\delta}{4+\gamma}\right)^{-k}\binom{\frac{1}{2}}{k}$$

for $|1248+67\gamma+245\delta|>67(4+\gamma)$

$$\sqrt{\frac{\left(1+\left(\frac{1}{2}\right)^2\delta\right)\left(1+\frac{1}{7}\left(\frac{1}{2}\right)^2(-128)\right)}{\left(1+\left(\frac{1}{2}\right)^2\gamma\right)\left(1+\frac{1}{343}\left(\frac{1}{2}\right)^2(-32)\right)}}=\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{67}\right)^k\left(-\frac{1248+67\gamma+245\delta}{4+\gamma}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

for $|1248+67\gamma+245\delta|<67(4+\gamma)$

$$\sqrt{\frac{\left(1+\left(\frac{1}{2}\right)^2\delta\right)\left(1+\frac{1}{7}\left(\frac{1}{2}\right)^2(-128)\right)}{\left(1+\left(\frac{1}{2}\right)^2\gamma\right)\left(1+\frac{1}{343}\left(\frac{1}{2}\right)^2(-32)\right)}}=\sqrt{-\frac{1248+67\gamma+245\delta}{67(4+\gamma)}}\sum_{k=0}^{\infty}\frac{(-67)^k\left(-\frac{1248+67\gamma+245\delta}{4+\gamma}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $|1248+67\gamma+245\delta|>67(4+\gamma)$

Definite integral after subtraction of diverging parts

$$\int_0^\infty \left(7\sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}}\sqrt{-1-\frac{\delta}{4}} - \left(7\sqrt{\frac{5}{67(4+\gamma)}}\sqrt{-\delta} + \frac{14\sqrt{\frac{5}{67(4+\gamma)}}\sqrt{-\delta}}{\delta}\right)\right) d\delta = -\frac{112}{3} i \sqrt{\frac{5}{67(4+\gamma)}} \approx -4.76698\, i$$

From:

$$7\sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}}\sqrt{-\frac{\delta}{4}-1}$$

For δ = -3

7 sqrt(5/(67 (1 + gamma /4))) sqrt(-1+(3)/4)

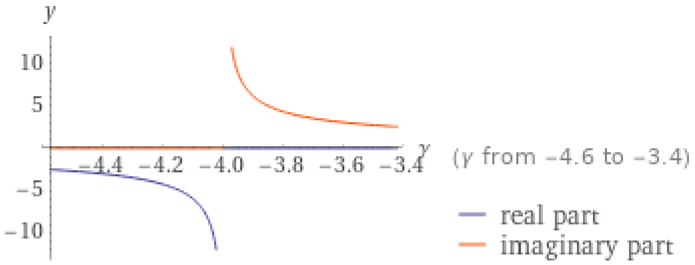
Input

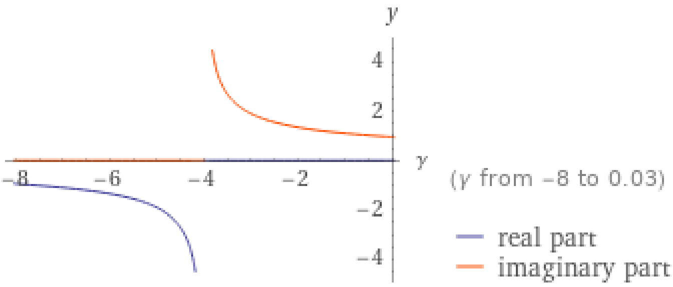
$$7\sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}}\sqrt{-1+\frac{3}{4}}$$

Exact result

$$\frac{7}{2} i \sqrt{\frac{5}{67}}\sqrt{\frac{1}{\frac{\gamma}{4}+1}}$$

Plots





Alternate form

$$7 i \sqrt{\frac{5}{67}} \sqrt{\frac{1}{\gamma+4}}$$

Roots

(no roots exist)

Series expansion at γ=-4

$$\frac{7 i \sqrt{\frac{5}{67}} \sqrt{\frac{1}{\gamma+4}} \sqrt{\gamma+4}}{\sqrt{\gamma+4}} + O\left((\gamma+4)^{11/2}\right)$$

(Puiseux series)

Series expansion at γ=0

$$\frac{7}{2} i \sqrt{\frac{5}{67}} - \frac{7}{16} i \sqrt{\frac{5}{67}}\, \gamma + \frac{21}{256} i \sqrt{\frac{5}{67}}\, \gamma^2 - \frac{35 i \sqrt{\frac{5}{67}}\, \gamma^3}{2048} + \frac{245 i \sqrt{\frac{5}{67}}\, \gamma^4}{65\,536} + O\left(\gamma^5\right)$$

(Taylor series)

Series expansion at γ=∞

$$7 i \sqrt{\frac{5}{67}} \sqrt{\frac{1}{\gamma}} - 14 i \sqrt{\frac{5}{67}} \left(\frac{1}{\gamma}\right)^{3/2} + 42 i \sqrt{\frac{5}{67}} \left(\frac{1}{\gamma}\right)^{5/2} - 140 i \sqrt{\frac{5}{67}} \left(\frac{1}{\gamma}\right)^{7/2} + 490 i \sqrt{\frac{5}{67}} \left(\frac{1}{\gamma}\right)^{9/2} + O\left(\left(\frac{1}{\gamma}\right)^5\right)$$

(Puiseux series)

Derivative

$$\frac{d}{d\gamma}\left(7 \sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}} \sqrt{-1+\frac{3}{4}}\right) = -\frac{7}{2} i \sqrt{\frac{5}{67}} \left(\frac{1}{\gamma+4}\right)^{3/2}$$

Indefinite integral

$$\int \frac{7}{2} i \sqrt{\frac{5}{67}} \sqrt{\frac{1}{1+\frac{\gamma}{4}}}\, d\gamma = \frac{14 i \sqrt{\frac{5}{67}}}{\sqrt{\frac{1}{\gamma+4}}} + \text{constant}$$

Series representations

$$7 \sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}} \sqrt{-1+\frac{3}{4}} = 7 \sqrt{-\frac{5}{4}} \sqrt{-1+\frac{20}{67(4+\gamma)}}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(-\frac{5}{4}\right)^{-k_1} \left(-1+\frac{20}{67(4+\gamma)}\right)^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} \quad \text{for } \left|1-\frac{20}{67(4+\gamma)}\right| > 1$$

$$7\sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}}\sqrt{-1+\frac{3}{4}} = 7\sqrt{-\frac{5}{4}}\sqrt{-1+\frac{20}{67(4+\gamma)}}$$

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\left(\frac{4}{5}\right)^{k_1}\left(-1+\frac{20}{67(4+\gamma)}\right)^{-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!} \quad \text{for } \left|1-\frac{20}{67(4+\gamma)}\right|>1$$

$$7\sqrt{\frac{5}{67\left(1+\frac{\gamma}{4}\right)}}\sqrt{-1+\frac{3}{4}} =$$

$$7\sqrt{z_0}^{\,2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(-\frac{1}{4}-z_0\right)^{k_1}\left(\frac{20}{67(4+\gamma)}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

Definite integral after subtraction of diverging parts

$$\int_0^{\infty}\left(\frac{7}{2}i\sqrt{\frac{5}{67}}\sqrt{\frac{1}{1+\frac{\gamma}{4}}}-\frac{7i\sqrt{\frac{5}{67}}}{\sqrt{\gamma}}\right)d\gamma = -28\,i\sqrt{\frac{5}{67}}\approx -7.64902\,i$$

From:

$$\frac{7}{2}i\sqrt{\frac{5}{67}}\sqrt{\frac{1}{\frac{\gamma}{4}+1}}$$

For γ = -3:

7/2 i sqrt(5/67) sqrt(1/(1 +(-3)/4))

Input

$$\frac{7}{2}i\sqrt{\frac{5}{67}}\sqrt{\frac{1}{1-\frac{3}{4}}}$$

i is the imaginary unit

Result

$$\frac{7\,i\,\sqrt{335}}{67}$$

Decimal approximation

$1.9122542764785356236183501957232557222554383558928776627750973 17\ldots\,i$

1.912254276….i

Polar coordinates

$$r = 7\sqrt{\frac{5}{67}} \text{ (radius)}, \quad \theta = \frac{\pi}{2} \text{ (angle)}$$

Polar forms

$$7\sqrt{\frac{5}{67}}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$$

Approximate form

$$7\sqrt{\frac{5}{67}}\, e^{(i\pi)/2}$$

From which, we obtain:

sqrt3/2 * 1/Pi^2 * ((7/2 i sqrt(5/67) sqrt(1/(1 +(-3)/4))))^-64

Input

$$\frac{\frac{\sqrt{3}}{2} \times \frac{1}{\pi^2}}{\left(\frac{7}{2}\, i\sqrt{\frac{5}{67}}\sqrt{\frac{1}{1-\frac{3}{4}}}\right)^{64}}$$

i is the imaginary unit

Result

$$\left(27\,189\,028\,279\,553\,414\,235\,049\,966\,267\,283\,185\,807\,800\,188\,603\,627\,566\,700\,161\sqrt{3}\right)/\left(56\,799\,523\,887\,961\,902\,662\,113\,954\,432\,447\,060\,046\,513\,345\,750\,572\,085\,427\,120\,327\,949\,523\,925\,781\,250\,\pi^2\right)$$

Decimal approximation

$8.4005916260011092573689959981962757722426360691573211190534\ldots \times 10^{-20}$

$8.400591626\ldots*10^{-20}$ = possible Axion mass

Property

$\left(27\,189\,028\,279\,553\,414\,235\,049\,966\,267\,283\,185\,807\,800\,188\,603\,627\,566\,700\,161\sqrt{3}\right)/\left(56\,799\,523\,887\,961\,902\,662\,113\,954\,432\,447\,060\,046\,513\,345\,750\,572\,085\,427\,120\,327\,949\,523\,925\,781\,250\,\pi^2\right)$ is a transcendental number

Alternate form

$$\left(27\,189\,028\,279\,553\,414\,235\,049\,966\,267\,283\,185\,807\,800\,188\,603\,627\,566\,700\,161\sqrt{3}\right)/\left(56\,799\,523\,887\,961\,902\,662\,113\,954\,432\,447\,060\,046\,513\,345\,750\,572\,085\,427\,120\,327\,949\,523\,925\,781\,250\,\pi^2\right)$$

Series representations

$$\frac{\sqrt{3}}{\left(\pi^2\left(\frac{1}{2}\, i\, 7 \sqrt{\frac{5}{67}} \sqrt{\frac{1}{1-\frac{3}{4}}}\right)^{64}\right)2} = \left(9\,223\,372\,036\,854\,775\,808 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k\, z_0^{-k}}{k!}\right) \Big/ \left(1\,219\,760\,487\,635\,835\,700\,138\,573\,862\,562\,971\,820\,755\,615\,294\,131\,238\,401\; i^{64}\, \pi^2 \sqrt{z_0}^{\,127} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{5}{67}-z_0\right)^k z_0^{-k}}{k!}\right)^{64} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4-z_0)^k\, z_0^{-k}}{k!}\right)^{64}\right) \quad \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{3}}{\left(\pi^2\left(\frac{1}{2}\, i\, 7 \sqrt{\frac{5}{67}} \sqrt{\frac{1}{1-\frac{3}{4}}}\right)^{64}\right)2} = \left(9\,223\,372\,036\,854\,775\,808 \exp\left(\pi \mathcal{A} \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(1\,219\,760\,487\,635\,835\,700\,138\,573\,862\,562\,971\,820\,755\,615\,294\,131\,238\,401\; i^{64}\, \pi^2 \exp^{64}\left(\pi \mathcal{A} \left\lfloor \frac{\arg\left(\frac{5}{67}-x\right)}{2\pi} \right\rfloor\right) \exp^{64}\left(\pi \mathcal{A} \left\lfloor \frac{\arg(4-x)}{2\pi} \right\rfloor\right) \sqrt{x}^{\,127} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{5}{67}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^{64} \left(\sum_{k=0}^{\infty} \frac{(-1)^k (4-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^{64}\right) \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{3}}{\left(\pi^2 \left(\frac{1}{2}\, i\, 7 \sqrt{\frac{5}{67}} \sqrt{\frac{1}{1-\frac{3}{4}}}\right)^{64}\right) 2} = \Bigg(9\,223\,372\,036\,854\,775\,808$$

$$\left(\frac{1}{z_0}\right)^{-32 \left\lfloor \arg\left(\frac{5}{67}-z_0\right)/(2\pi)\right\rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi)\rfloor - 32 \lfloor \arg(4-z_0)/(2\pi)\rfloor}$$

$$z_0^{-127/2-32 \left\lfloor \arg\left(\frac{5}{67}-z_0\right)/(2\pi)\right\rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi)\rfloor - 32 \lfloor \arg(4-z_0)/(2\pi)\rfloor}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k\, z_0^{-k}}{k!}\Bigg) \Big/$$

$$\Bigg(1\,219\,760\,487\,635\,835\,700\,138\,573\,862\,562\,971\,820\,755\,615\,294\,131\,238\,401$$

$$i^{64}\, \pi^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{5}{67}-z_0\right)^k z_0^{-k}}{k!}\right)^{64} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4-z_0)^k\, z_0^{-k}}{k!}\right)^{64}\Bigg)$$

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

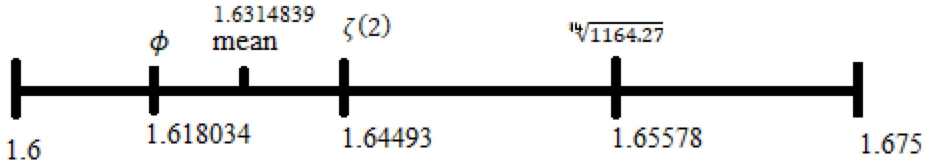
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

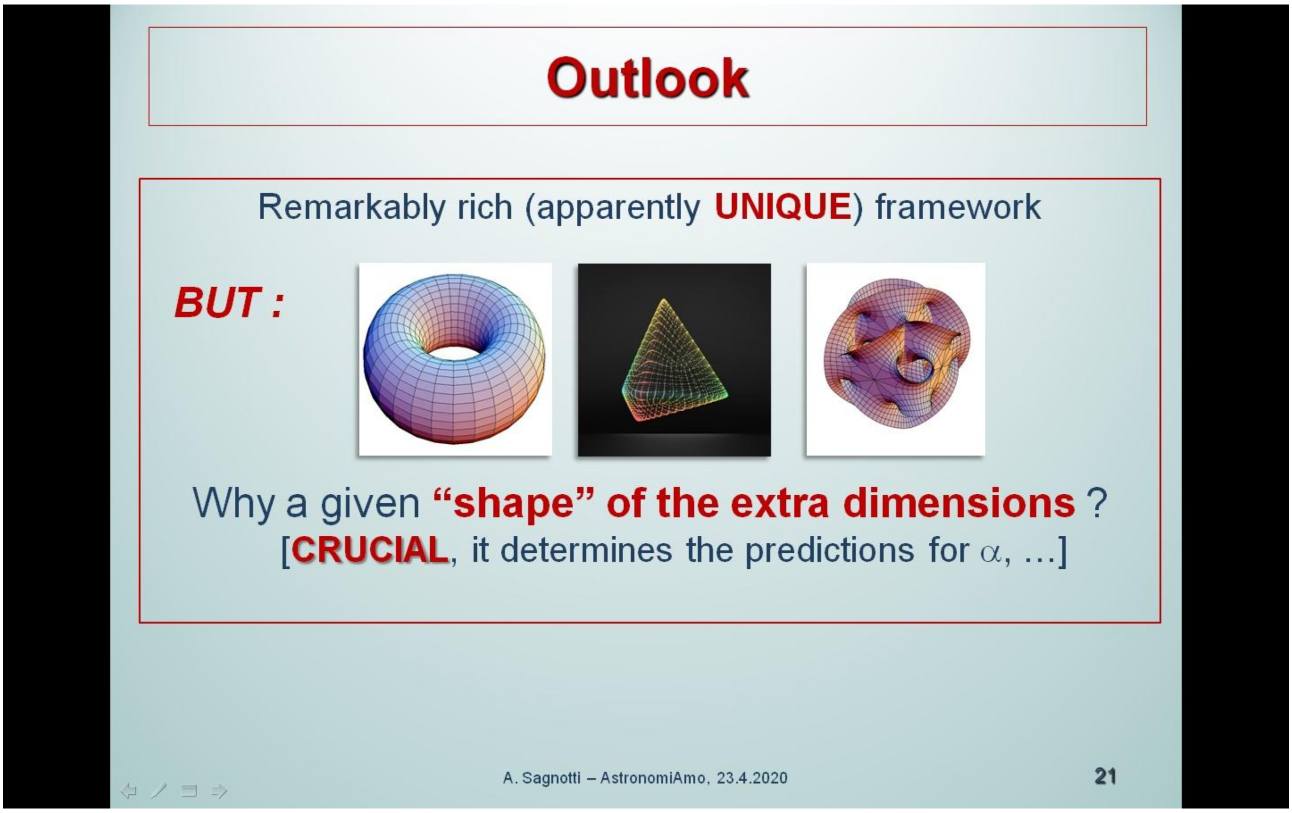
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



Outlook

Remarkably rich (apparently **UNIQUE**) framework

BUT :

Why a given **“shape” of the extra dimensions** ?

[**CRUCIAL**, it determines the predictions for α, …]

A. Sagnotti – AstronomiAmo, 23.4.2020

21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for α”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

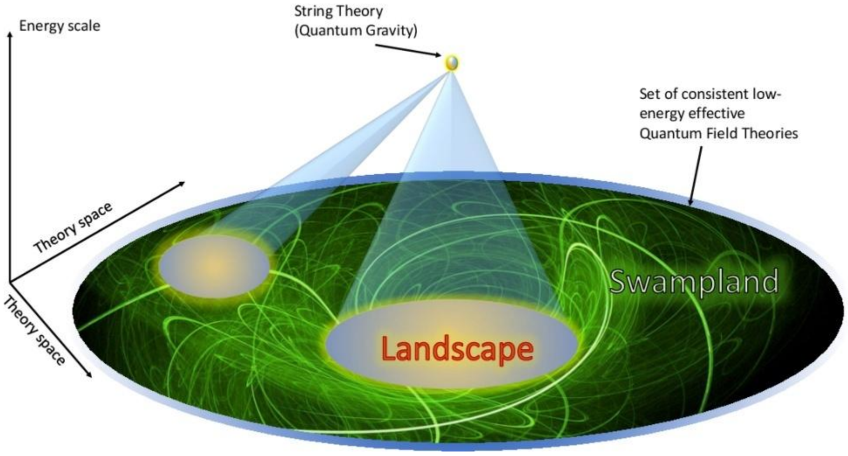
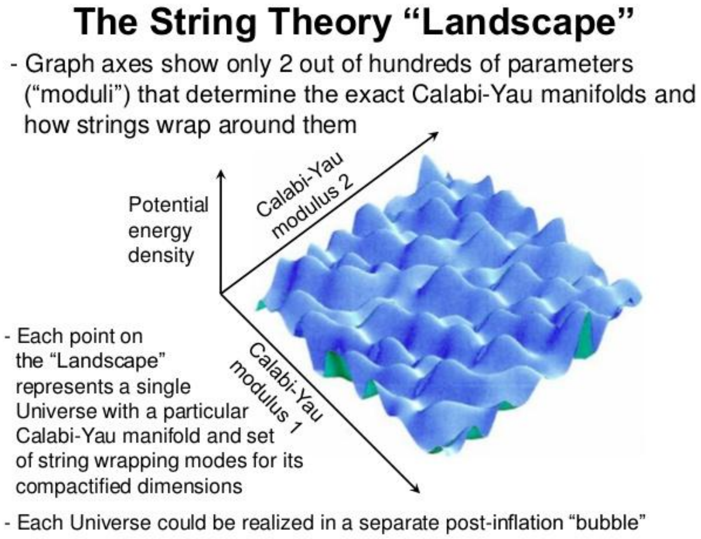


Fig. 1



The String Theory "Landscape"

- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

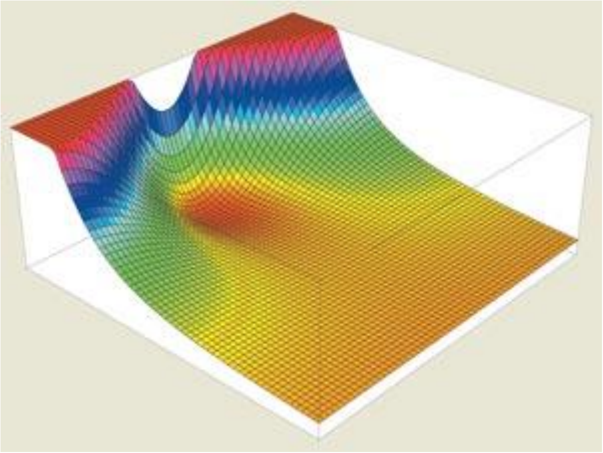


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

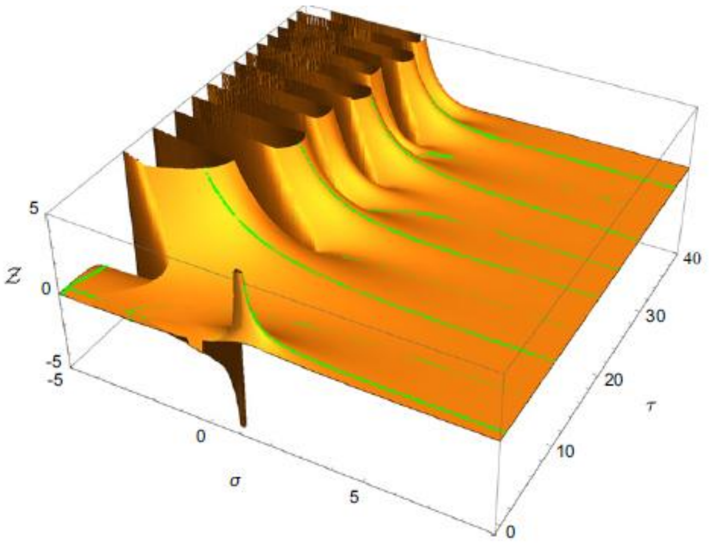


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + \mathrm{i}\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

$$\text{with } k = \ldots, -2, -1, 0, 1, 2, \ldots.$$

we obtain:

2Pi/(ln(2))

Input:

$$2\times\frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958
…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\,i\,\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\,i\,\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\,dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4\,i\,\pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\,\pi\log(2)\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

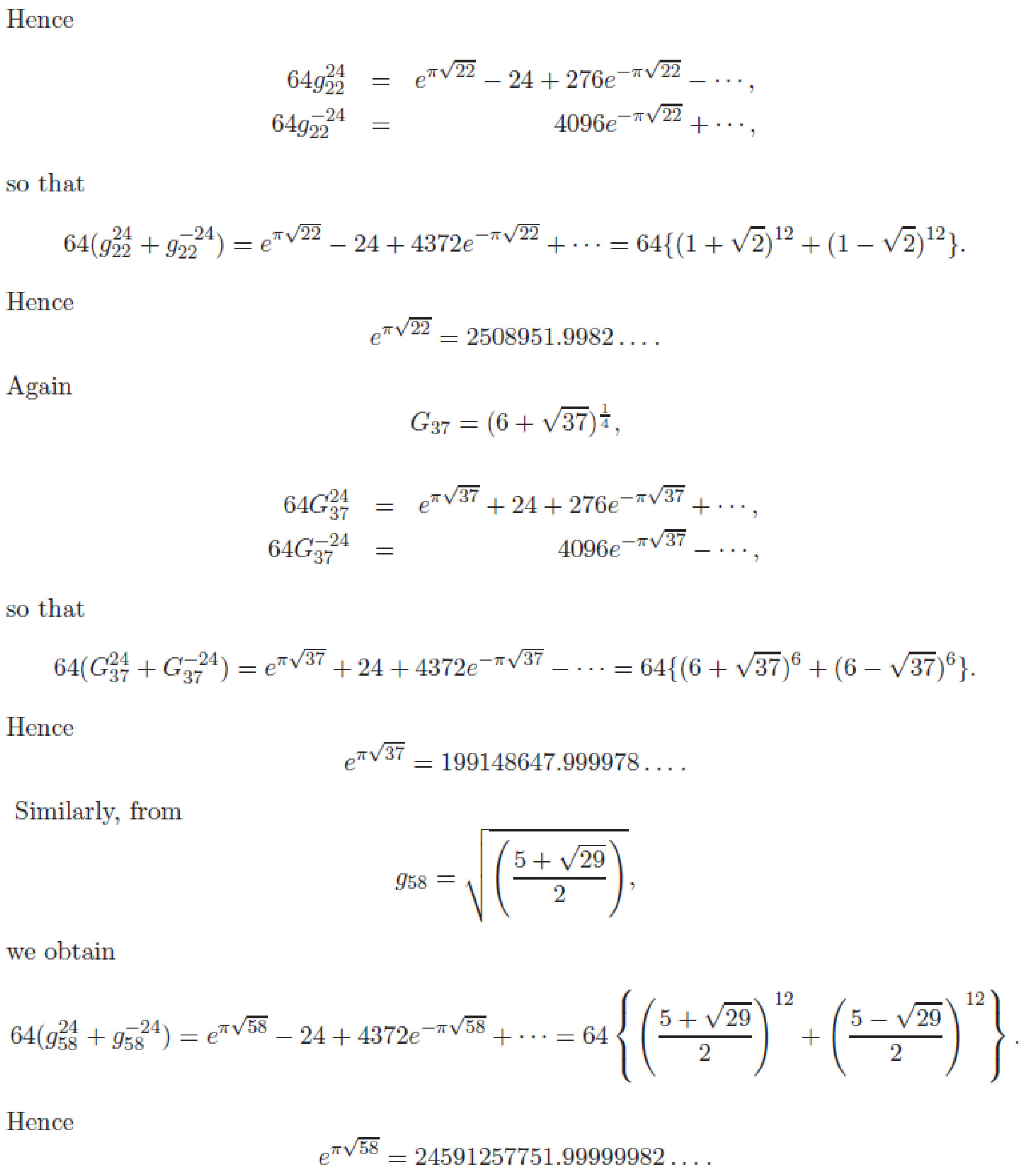
1.6449340668482264364724151666460251892189499012067984377355582293
…

$1.6449340668\ldots. = \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:



Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023...

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}$$

Minimal polynomial

$$256x^8 + 95\,744x^7 - 3\,248\,750\,080x^6 - 914\,210\,725\,504x^5 + 15\,498\,355\,554\,921\,184x^4 + 2\,911\,478\,392\,539\,914\,656x^3 - 32\,941\,144\,911\,224\,677\,091\,680x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Bigg/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$
\begin{aligned}
&27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi= \\
&\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+\right. \\
&\qquad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}- \\
&\qquad \left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/ \\
&\quad\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi= \\
&\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+\right. \\
&\qquad 108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+ \\
&\qquad 2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}- \\
&\qquad \left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2\sqrt{5}-z_0\right)^{k}z_0^{-k}}{k!}\right)\Big/ \\
&\quad\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}\right)\right)
\end{aligned}
$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544 7023

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Bigg/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+\right.$$

$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}-$$

$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$

$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+\right.$$

$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+$$

$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}-$$

$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2\sqrt{5}-z_0\right)^{k}z_0^{-k}}{k!}\right)\Big/$$

$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.6438185685849862799902301317036810054185756873505184804834183124

...

1.64381856858…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\boxed{\begin{array}{l}\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + \\ \quad 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - \\ \quad 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + \\ \quad 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05\end{array}}}$$

Minimal polynomial

$$\begin{array}{l}256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - \\ \quad 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + \\ \quad 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - \\ \quad 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041\end{array}$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2 i \pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4 i \pi)/15} \sqrt[15]{\phi + 27 \left(-2 + 2 \sqrt{1093} - \frac{\sqrt{10 - 2 \sqrt{5}} - 2}{2 \left(\sqrt{5} - 1\right)}\right)} \approx 1.0999 + 1.2216 i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\phi + 27 \left(-2 + 2 \sqrt{1093} - \frac{\sqrt{10 - 2 \sqrt{5}} - 2}{2 \left(\sqrt{5} - 1\right)}\right)} \approx 0.5080 + 1.5634 i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\phi + 27 \left(-2 + 2 \sqrt{1093} - \frac{\sqrt{10 - 2 \sqrt{5}} - 2}{2 \left(\sqrt{5} - 1\right)}\right)} \approx -0.17183 + 1.63481 i$$

Series representations

$$\sqrt[15]{27 \left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2 \sqrt{5}} - 2}{\left(\sqrt{5} - 1\right) 2}\right) + \phi} =$$

$$\frac{1}{\sqrt[15]{2}} \left(\left(\left(162 - 108 \sqrt{1093} - 2 \phi - 108 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108 \sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2 \phi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27 \sqrt{9 - 2 \sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(9 - 2 \sqrt{5}\right)^{-k}\right) \Big/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right) \hat{\ } (1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\Bigg(\Bigg(\Bigg(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$

$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$

$$27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\Bigg)\Bigg/$$

$$\Bigg(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\Bigg)^\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\Bigg(\Bigg(\Bigg(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$

$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$

$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$

$$27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\Bigg)\Bigg/$$

$$\Bigg(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\Bigg)\Bigg)^\wedge(1/15)\Bigg)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-\,2\,C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\, e^{-\,2\,A} \;+\; \frac{h^2}{16(p\,+\,1)}\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$

$1.6272016\ldots * 10^{-6}$

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

from which:

$\frac{1}{4096}e^{-6C+\phi}$ = 1.6272016… * 10^-6

0.000244140625 $e^{-6C+\phi}$ = $e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625}\, e^{-6C+\phi} = \frac{1}{0.000244140625}\, e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}$$

Result:

$$0.00666501785\ldots$$

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

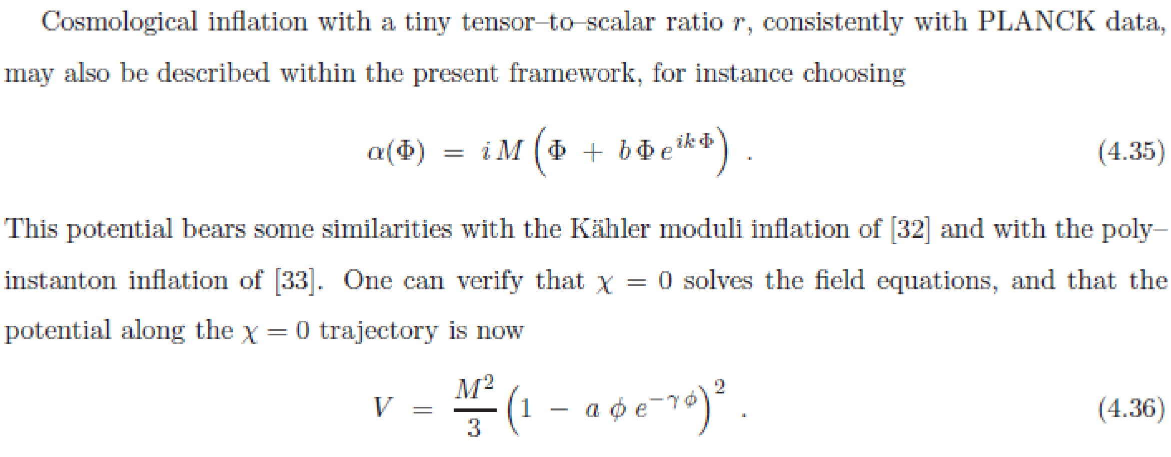
0.99040073.... result very near to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity
E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:



Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi)\;=\;i\,M\left(\Phi\;+\;b\,\Phi\,e^{ik\,\Phi}\right)\;. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi=0$ solves the field equations, and that the potential along the $\chi=0$ trajectory is now

$$V\;=\;\frac{M^2}{3}\left(1\;-\;a\;\phi\;e^{-\gamma\,\phi}\right)^2\;. \tag{4.36}$$

We analyzing the following equation:

$$V\;=\;\frac{M^2}{3}\left(1\;-\;a\;\phi\;e^{-\gamma\,\phi}\right)^2\;.$$

$$\phi = \varphi - \frac{\sqrt{6}}{k} ,$$

$$a = \frac{b\gamma}{e} < 0 , \qquad \gamma = \frac{k}{\sqrt{6}} < 0 .$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V = \frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V = \frac{1}{3}\left(0.0814845\, b + 1\right)^2 M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\, M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2 M^2$$

$$V = 0.00221324\left(b^2 M^2 + 24.5445\, b\, M^2 + 150.609\, M^2\right)$$

$$-0.00221324\, b^2 M^2 - 0.054323\, b\, M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\, b^2 M^2 + 0.054323\, b\, M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2 M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\, b^2 M^2 + 0.054323\, b\, M^2 + 0.333333\, M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b, M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b, M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}\,M^2\left(1-\frac{(b\;2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

$$\text{for } b=-\frac{226\,802\,245}{18\,480\,874}$$

$$\min\left\{\frac{1}{3}\,M^2\left(1-\frac{(b\;2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

$$\text{for } M=0$$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

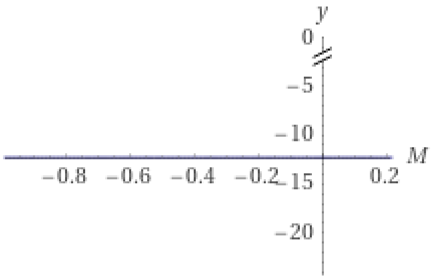
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

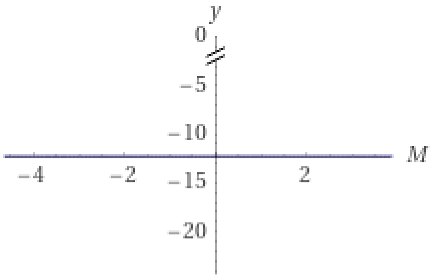
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(M from −1 to 0.2)



(M from −4.6 to 3.9)

Alternate form assuming M is real:

−12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2 - 1.21228\times 10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4} - 12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

-12.2723

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}\ -0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times 10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}\ -0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}\ -0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} - -12.2723 \right) dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}\,(0.0814845\, b + 1)^2\, M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

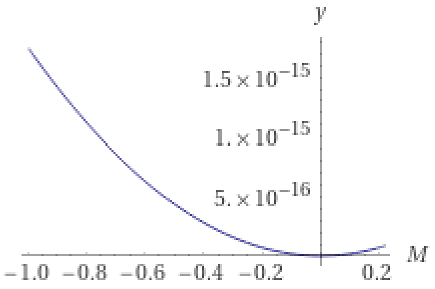
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

0

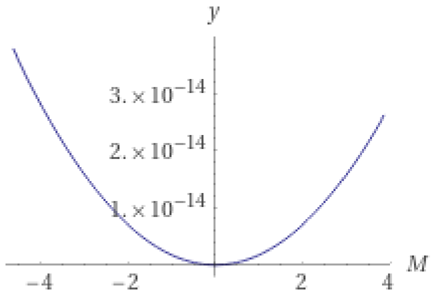
Plots: **(possible mathematical connection with an open string)**



(M from -1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(M from -4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$$O\left(M^{62194}\right)$$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$-4.388513449474645453489707833780880208333333333333333333333\ldots\times10^{-16}$

-4.38851344947*10^{-16}

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.02162151915943272558353253404940833333333333333333333333333\ldots\times10^{-17}$

7.021621519159*10^{-17}

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.5798648418108723632562948201611168 75\times10^{-14}$

1.57986484181*10^{-14}

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)])))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\ \hat{}\ (1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\ \sqrt[4]{3}\ e\log^{3/2}(3)}\right)$$

log(x) is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck’s Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck’s Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck’s Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

E_P * l_P = $(1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306\times 10^{61})\times 5.729475}{10^{35}}$$

Result:

$1\,042\,939\,771\,935\,000\,000\,000\,000\,000$

Scientific notation:

$1.042939771935\times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

E_P * l_P^2 / l_P = $(5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

$$5.975498\times 10^{-8}\times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542200500133\ldots\times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

References

On the Zeros of the Davenport Heilbronn Function
S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

A. A. Karatsuba, "On the zeros of arithmetic Dirichlet series without Euler product," Izv. Ross. Akad. Nauk, Ser. Mat. 57 (5), 3–14 (1993)

Connecting the Extremes: A Story of Supermassive Black Holes and Ultralight Dark Matter - *Hooman Davoudiasl, Peter B. Denton and Julia Gehrlein* - arXiv:2109.01678v1 [astro-ph.CO] 3 Sep 2021

Ramanujan Manuscript Book 1 – *Srinivasa Ramanujan*

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity
E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015